Computation of Tokamak Edge Turbulence

techniques and typical results

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Outline

- Basics of Low-frequency Drift Dynamics

  — low frequency basics, energy transfer in turbulence, equilibrium

- Methods

  — numerical techniques, mathematical treatment of magnetic geometry

- Electromagnetic Nonlinear Character

  — energy transfer, nonlinear saturation, turbulence is not a collection of instabilities

- Sheared ExB Flows

  — basic mechanisms, self consistency, toroidal compression

- Gyrofluid Edge Turbulence

  — self consistent evolution of MHD equilibrium

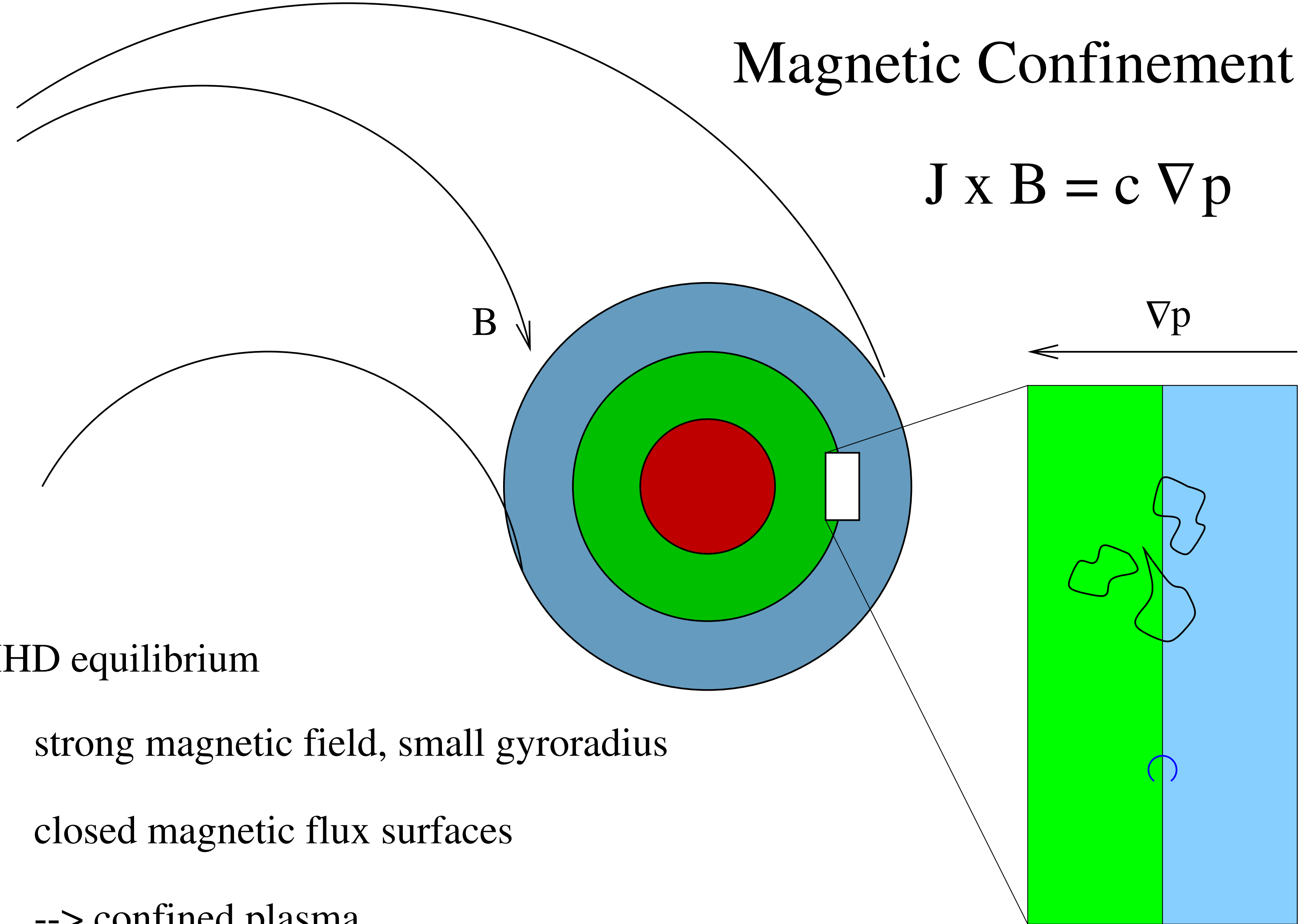
  — scale separation, role of gyro-Bohm scaling

  — global burst behaviour, edge/SOL interface, edge to SOL causality

- Some Important Lessons

Magnetic Confinement

$$J \times B = c \nabla p$$



MHD equilibrium

strong magnetic field, small gyroradius

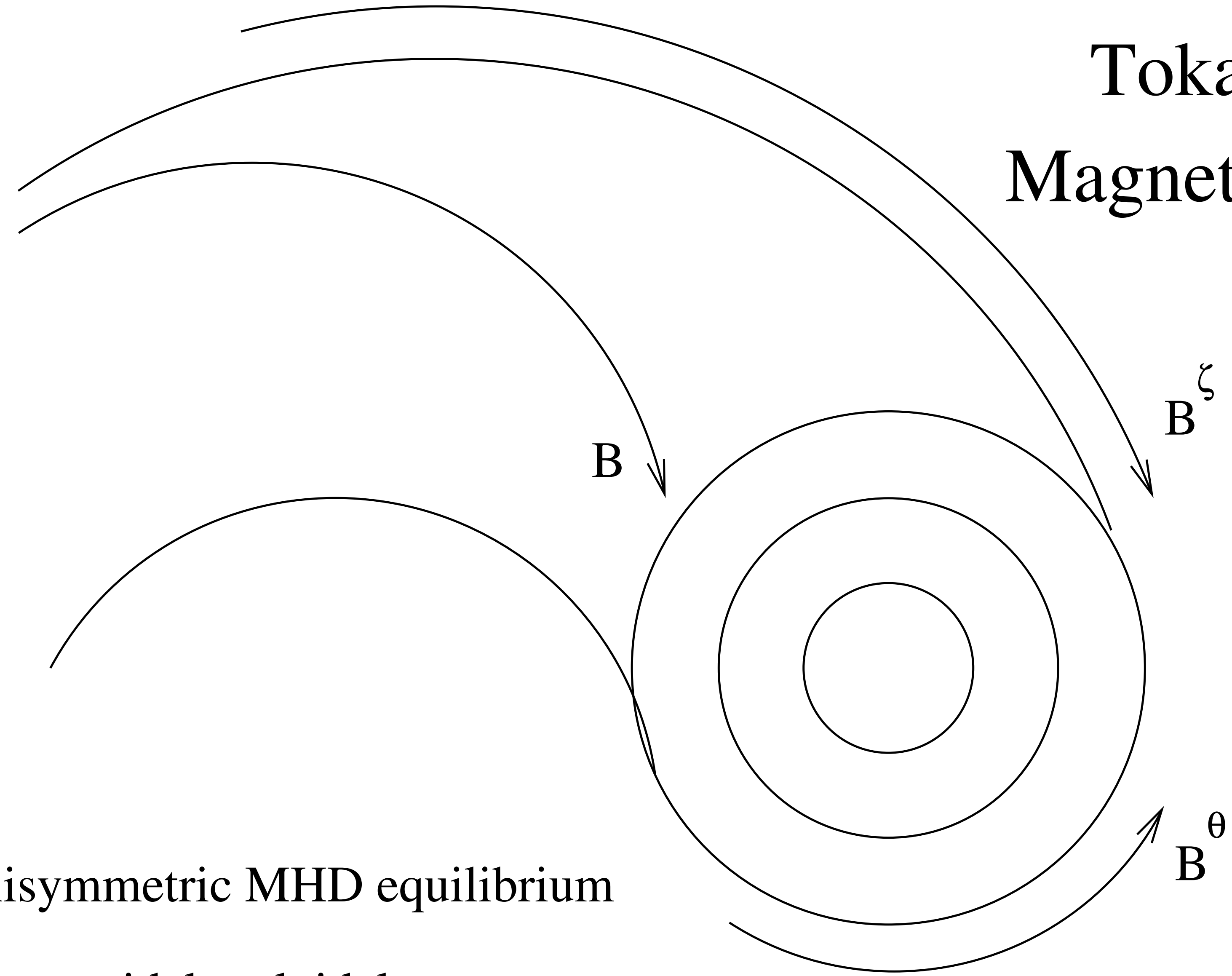
closed magnetic flux surfaces

--> confined plasma

however . . . turbulence --> losses

eddies, few gyroradii

Tokamak Magnetic Field



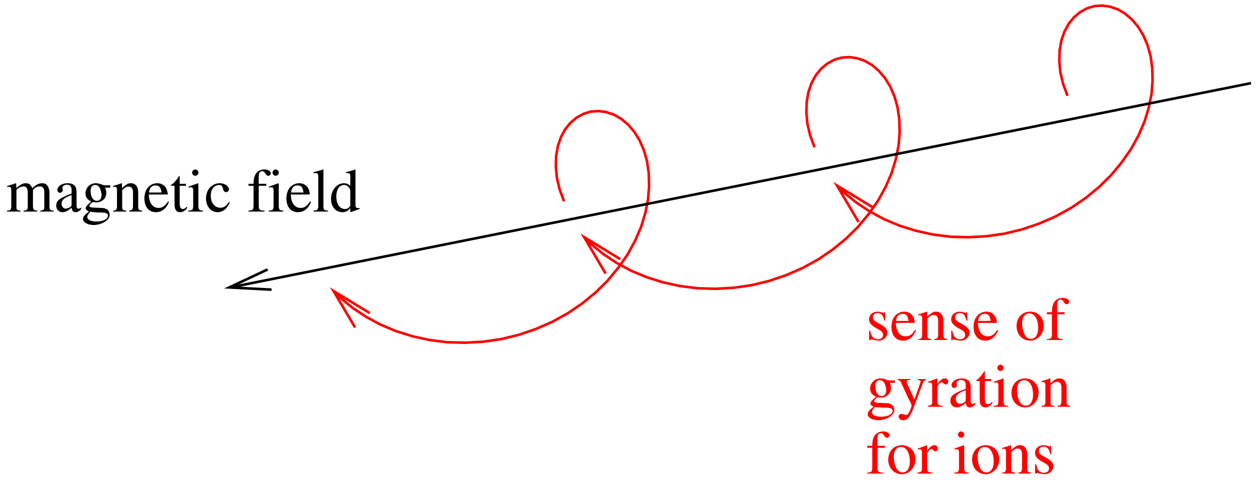
axisymmetric MHD equilibrium

toroidal, poloidal components

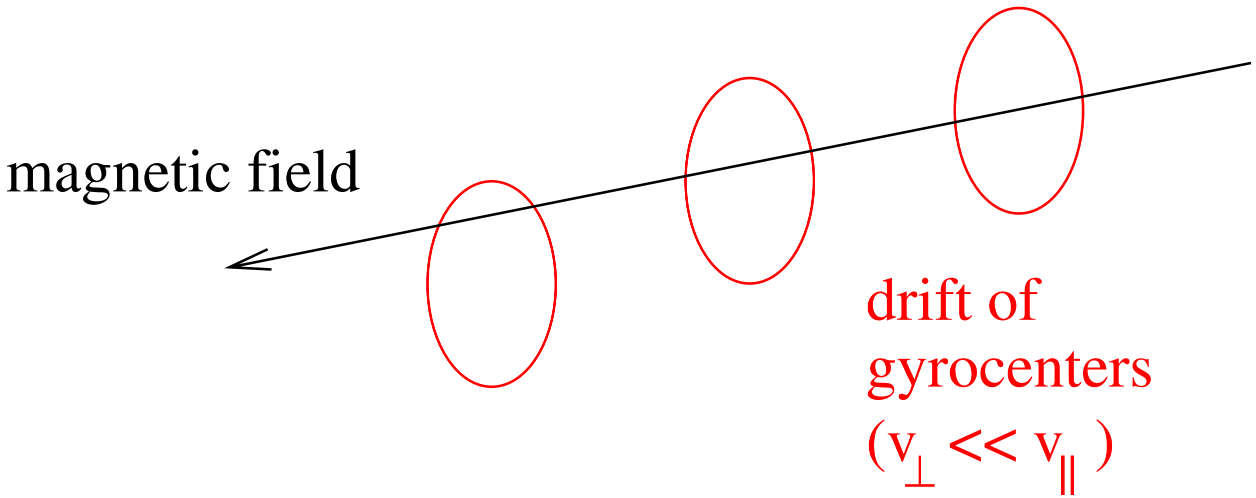
mainly toroidal

ratio of components --> pitch parameter ‘‘q’’ B^{ζ} / B^{θ}

Low Frequency Drift Motion



general



low frequencies

$\omega << \Omega$

v-space details: ‘‘gyrokinetic’’

few moments: ‘‘gyrofluid’’

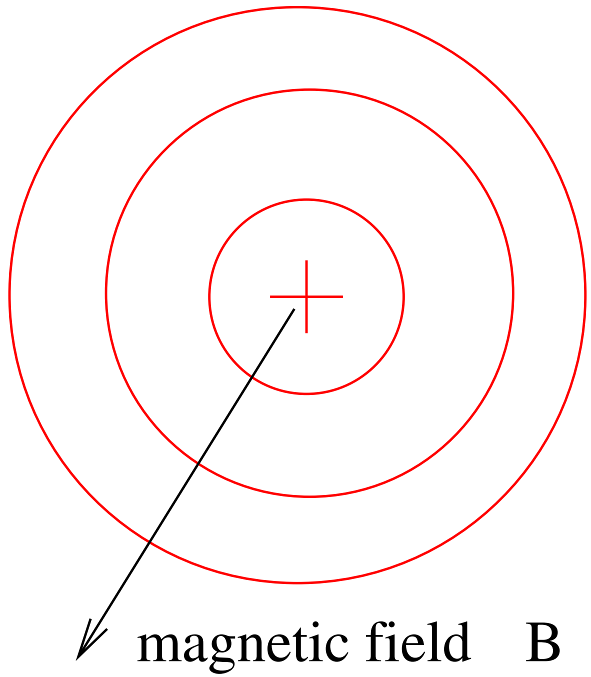
Low Pressure (Beta) Dynamics

low ‘‘beta’’

$p << B^2/8\pi$

low frequencies

$\omega << k_\perp v_A$



magnetic field B

pressure disturbance $\tilde{p}$

magnetic disturbance $\tilde{B}$
(parallel to B)

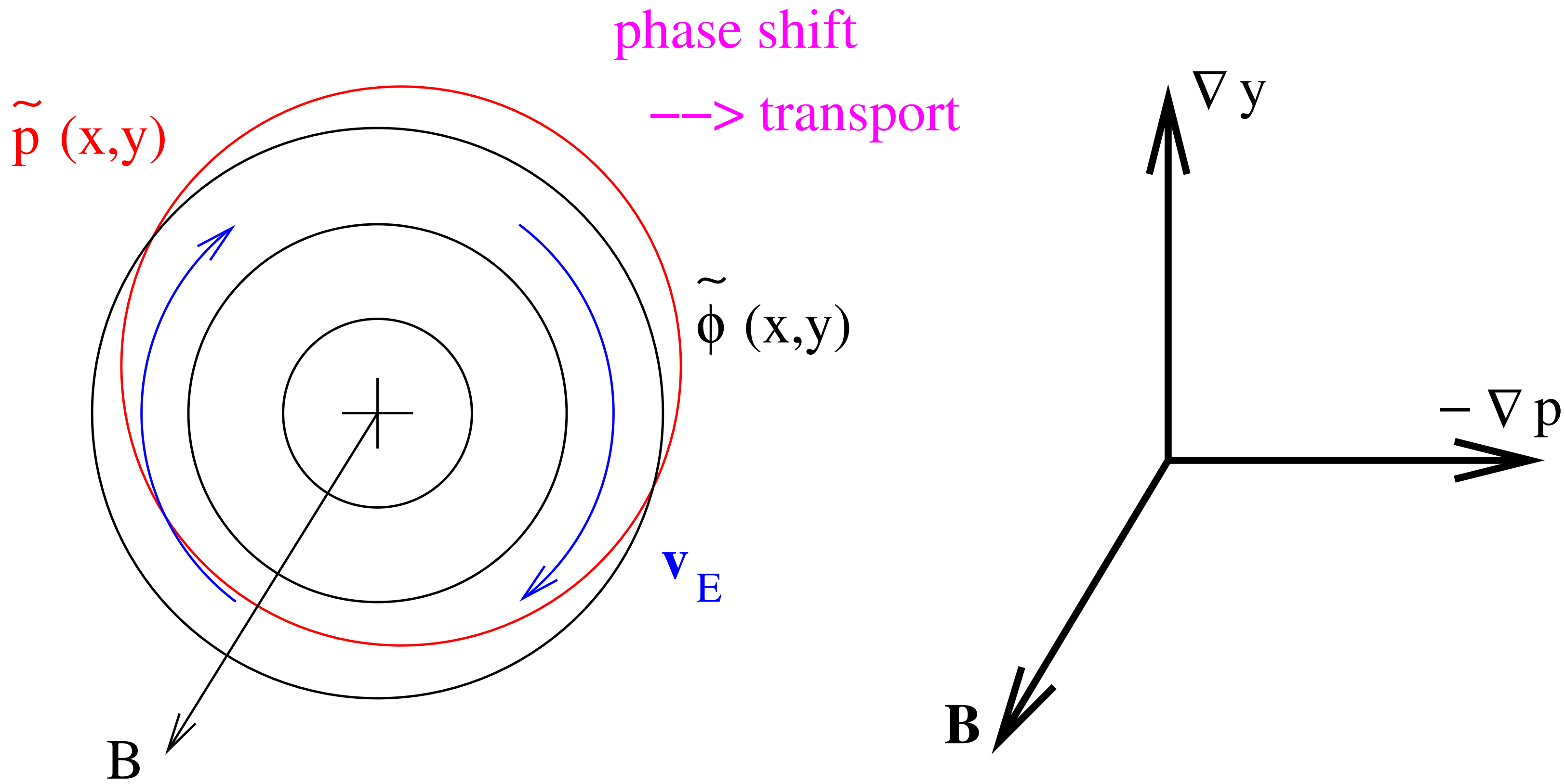
‘‘flute mode’’
vortices/filaments

$k_{||} << k_\perp$

--> strict perpendicular force balance $\nabla(\tilde{p} + 4\pi\, B\tilde{B}) \sim 0$

$\omega \sim k_{||} v_A$ --> electromagnetic parallel dynamics

Sense of Coordinate Geometry



computations: align coordinates to magnetic field (sheared, curved)

(only one contravariant component of B is nonvanishing)

(nonorthogonal, takes advantage of slowly varying B)

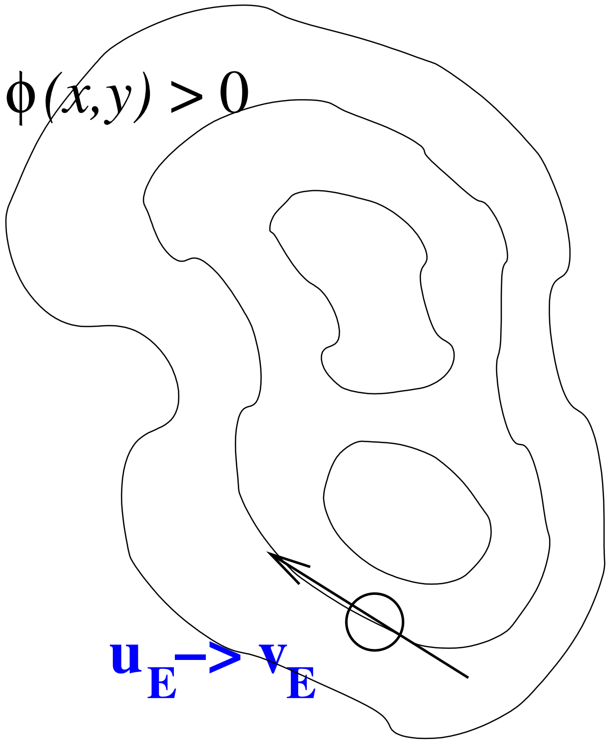
(S Cowley et al Phys Fluids B 1991, B Scott Phys Plasmas 1998, 2001)

ExB Drift at Finite Gyroradius

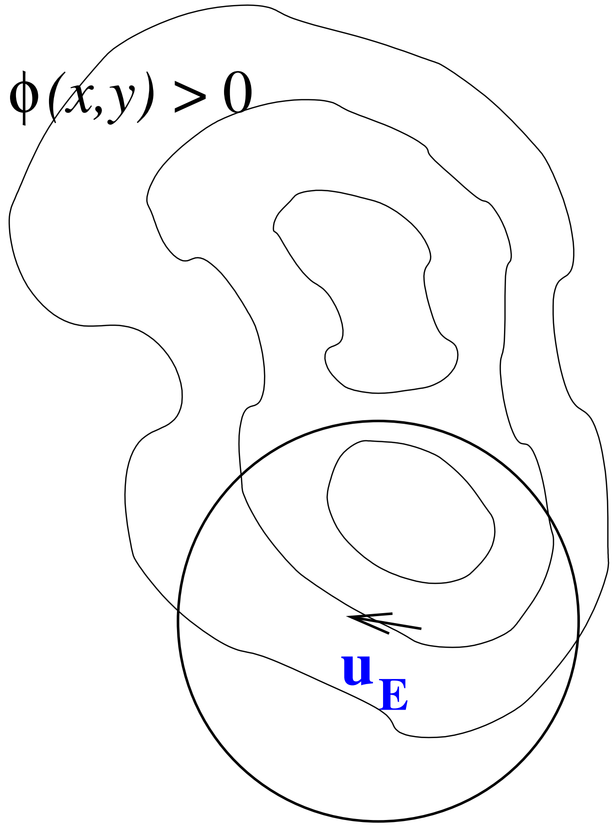
$$v_E = \frac{c}{B^2} B \times \nabla \phi$$

$$u_E = \frac{c}{B^2} B \times \nabla J_0 \phi$$

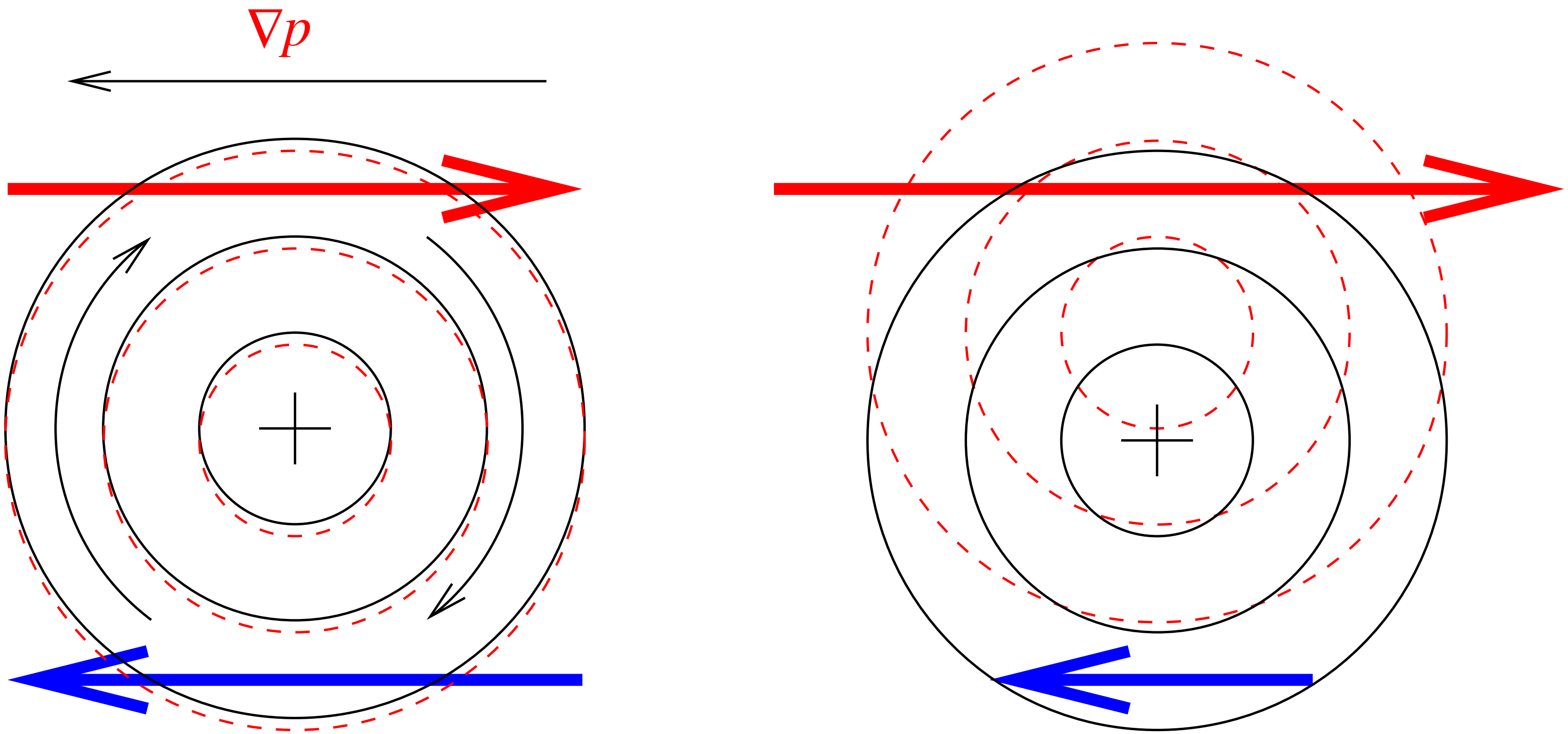
$k\rho \ll 1$



$k\rho \sim 1$



Phase Shifts and Transport



p and phi in phase

--> no net transport

phase shift --> net transport

phase shift --> net transport down gradient

--> free energy drive

Role of Parallel Forces on Electrons

equation of motion for electrons parallel to B

$$n_e e \left(\frac{1}{c} \dot{A}_\parallel + \nabla_\parallel \phi + \eta_\parallel J_\parallel \right) = \nabla_\parallel p_e + \text{inertia}$$

Alfven (MHD) coupling

adiabatic (fluid compression) coupling

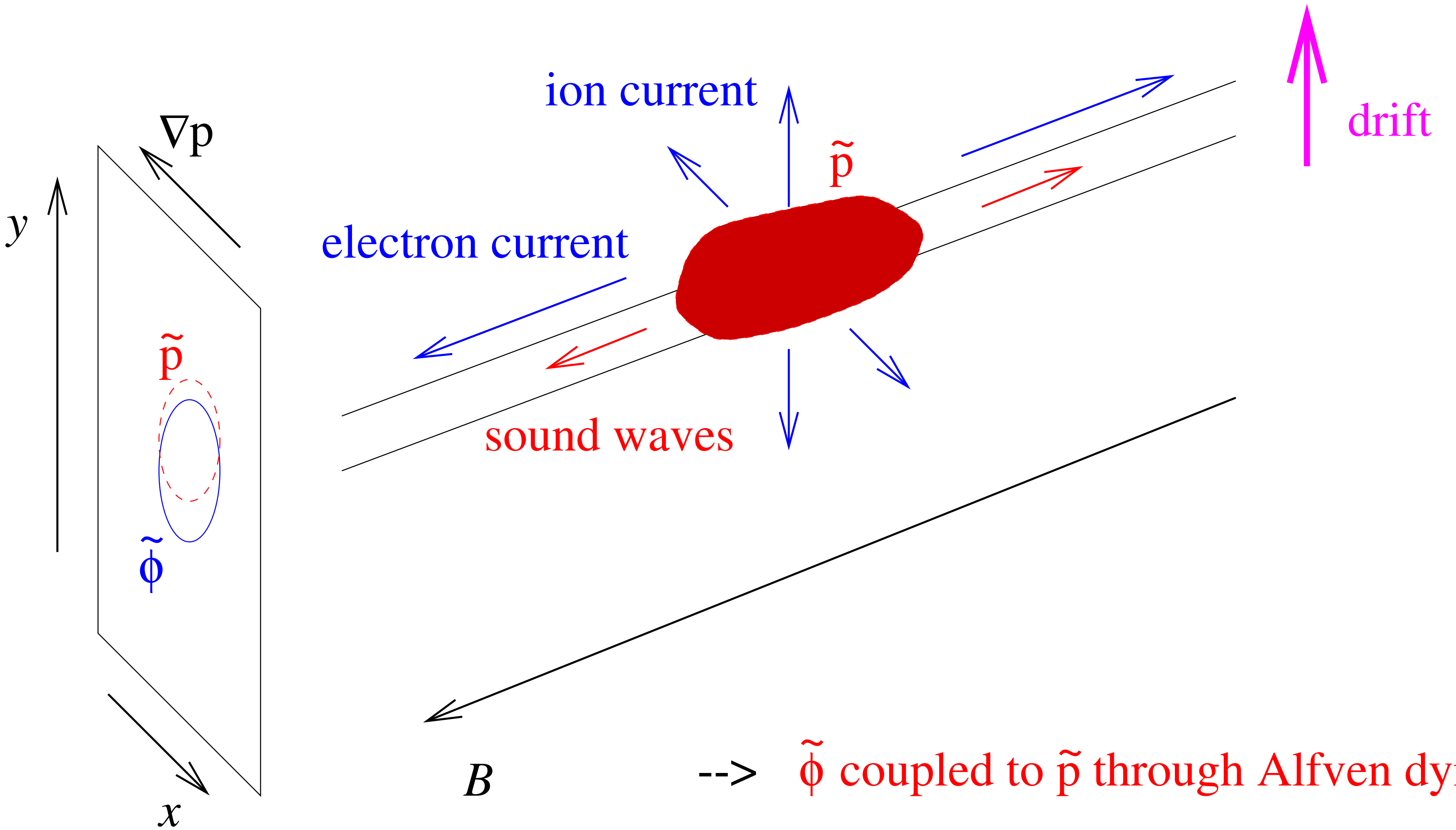
a ‘‘two fluid’’ effect

static balance of gradients --> ‘‘adiabatic electrons’’

general: response of currents to static imbalance

controls possible phase shifts $\tilde{p}_e$ <--> $\tilde{\phi}$

Drift (Alfven) Wave Dynamics



--> $\tilde{\phi}$ coupled to $\tilde{p}$ through Alfven dynamics

--> $\tilde{\phi}$ continually excites $\tilde{p}$ in the gradient

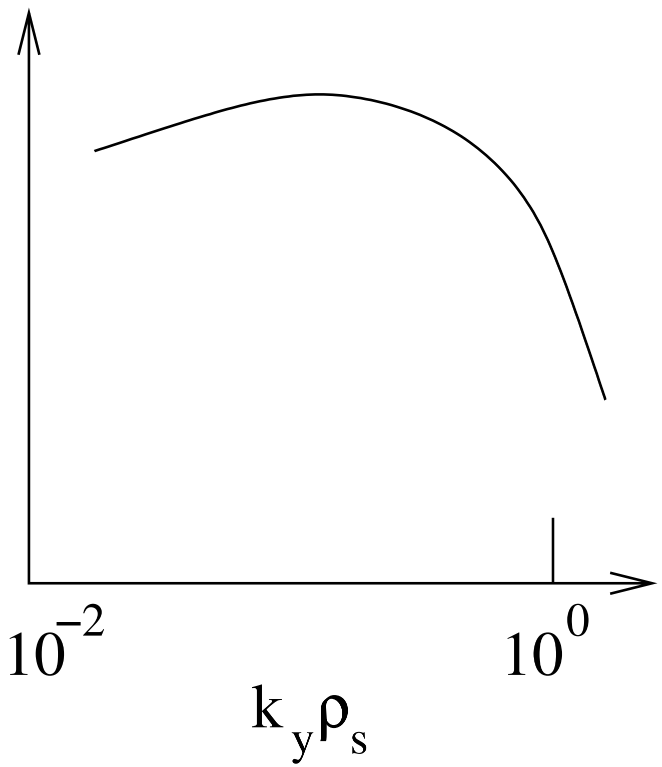
--> structure drifts

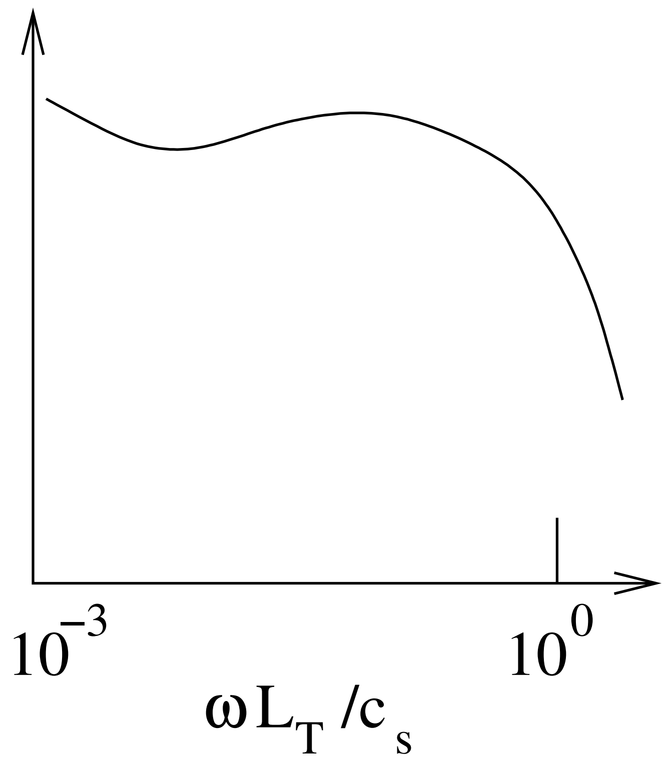
(M Wakatani A Hasegawa Phys Fluids 1984)

(B Scott Plasma Phys Contr Fusion 1997)

Scales of Motion

broad range of both time and space scales –– to ion gyroradius





slowest time scale reflect flow/equilibrium component

for equal temperatures, space scale range includes ion gyroradius

high resolution, long runs (> 1000 "gyro–Bohm" times) are necessary

(B Scott Plasma Phys Contr Fusion 2003)

Numerical Methods

- nonlinearities have the form of brackets

$$\frac{\partial f}{\partial t} + [\psi, f]_{xy} + \cdots = 0 \qquad \text{with} \qquad [\psi, f]_{xy} = \frac{\partial \psi}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial f}{\partial x}\frac{\partial \psi}{\partial y}$$

- spatial discretisation:
  centred-diff for linear terms, Arakawa (J Comput Phys 1966) scheme for brackets
  ◦ basic properties of bracket satisfied to machine accuracy

$$[\phi, f]_{xy} = \frac{1}{3}\left(J^{++} + J^{+\times} + J^{\times+}\right)$$

- temporal discretisation:
  "stiffly stable" form (Karniadakis et al J Comput Phys 1991), stable for waves
  ◦ both sides expanded $\Longrightarrow$ all mixed terms in Taylor expansion present
  ◦ one evaluation per time step
  ◦ tested on turbulence and coherent vortices (Naulin and Nielsen, SIAM J Math 2003)

$$\frac{\partial f}{\partial t} = S \qquad \text{with} \qquad \sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{\Delta t} = \sum_{j=1}^{3} \beta_j \, S_j$$

(details 1: origin of brackets)

- basic structure of gyrocenter continuity equation (similar to gyrokinetic equation)

$$\frac{\partial n}{\partial t} + \nabla\phi \cdot \frac{c\mathbf{F}}{B^2} \cdot \nabla n + B\nabla_\parallel \frac{n u_\parallel}{B} + \nabla \log B^2 \cdot \frac{c\mathbf{F}}{B^2} \cdot \left(n\nabla\phi + \frac{1}{e}\nabla p \right) = 0$$

- define bracket

$$\nabla n \cdot \frac{c\mathbf{B}}{B^2} \times \nabla\phi = \nabla\phi \cdot \frac{c\mathbf{F}}{B^2} \cdot \nabla n \equiv [\phi, n] \qquad \text{using} \qquad \mathbf{F} = \epsilon \cdot \mathbf{B}$$

- local approximations: use $L_\perp k_\perp \gg 1$ ordering
  - linearise everywhere except in bracket-structure quadratic nonlinearities

$$\frac{\partial n}{\partial t} + [\phi, n] + n_0 B\nabla_\parallel \frac{u_\parallel}{B} + \left[\log B^2, \left(n_0\phi + \frac{T_0}{e} n + \frac{n_0}{e} T \right) \right] = 0$$

- adjust brackets to be divergence-free structures (preserves energetics)

(details 2: each structure)

- brackets (see geometry, below), simplified form
  - do the quantity in parentheses with Arakawa's discretisation

$$[\phi, f] = \frac{c}{B_0}\frac{1}{r}\left(\frac{\partial\phi}{\partial r}\frac{\partial f}{\partial\theta} - \frac{\partial\phi}{\partial\theta}\frac{\partial f}{\partial r}\right)$$

- parallel derivatives (see geometry, below), simplified form (linear term shown)
  - do these via centred differences (or 4th order if you wish)

$$B\nabla_{\parallel} f = \frac{B_0}{2\pi R_0}\left(\frac{\partial f}{\partial\zeta} + \frac{1}{q}\frac{\partial f}{\partial\theta}\right)$$

- dissipation terms are simple, e.g.,

$$\frac{1}{c}\frac{\partial A_{\parallel}}{\partial t} = \cdots - \eta_{\parallel} J_{\parallel} \qquad \text{or} \qquad \frac{1}{2}\frac{\partial T_{\parallel}}{\partial t} = \cdots - \frac{\nu}{3\eta_0}\left(T_{\parallel} - T_{\perp}\right)$$

Representation of Tokamak Geometry

- flux coordinates with nested flux surfaces (S Hamada, 1958, Nucl Fusion 1962)
  ○ surface label minor radius r, poloidal/toroidal angles θ, ζ periodic on unit torus

$$\mathbf{B} \cdot \nabla r = 0 \qquad \mathbf{B} \cdot \nabla \theta = B_0 / 2\pi q R_0 \qquad \mathbf{B} \cdot \nabla \zeta = B_0 / 2\pi R_0$$

- take advantage of $k_\parallel \ll k_\perp$ and align the coordinates to $\mathbf{B}$ — note q is $q(r)$

$$x = r/a \qquad y = q\theta - \zeta \qquad s = \theta$$

- this ensures that only one contravariant component is nonzero (here: B^s)
  ○ coarse resolution is allowed in that direction (dimension)
  ○ very high resolution, necessary for both $\mathbf{k}_\perp$ dimensions, becomes feasible
- typically MHD $\leftrightarrow$ turbulence crosstalk requires 500 or more toroidal modes
- main caveat: global consistency in θ boundary conditions

$$f(x, y + q, s + 1) = f(x, y, s) \qquad \text{ensures} \qquad k_\parallel q R = m - nq$$

More Work on the Coordinates

- main issue is deformation: large values of $g^{xy} \rightarrow$ extra numerical dissipation

- solution: different coordinate system on each "drift plane" $s = s_k =$ constant

$$x = r/a \qquad\qquad y_k = q(\theta - s_k) - \zeta - \Delta\alpha_k \qquad\qquad s = \theta$$

- non-zero $\nabla r \cdot \nabla\theta$ and $\nabla r \cdot \nabla\zeta$, choose

$$\alpha_k = q s_k + \Delta\alpha = \alpha_k(r) \qquad\qquad \frac{\partial}{\partial r}\Delta\alpha = \left(g^{rr}\right)^{-1}\left(q g^{r\theta} - g^{r\zeta}\right)$$

- this makes $g_k^{xy} = 0$ at $s = s_k$
  - retaining global field aligning, local orthogonality, exactly

- this "shifted metric" technique is required to treat anything with "slab character"
  - e.g., shear Alfvén turbulence component, global MHD such as tearing

- carry angle periodicity through, exactly, to obtain angle boundary conditions

(details 1: boundary conditions on angles)

- coordinates defined as

$$x = r/a \qquad y_k = q(\theta - s_k) - \zeta - \Delta\alpha_k \qquad s = \theta$$

- already satisfy toroidal periodicity
  ◦ changing y_k holding x, s constant is same as changing ζ holding r, θ constant

$$f(r, \theta, \zeta + 1) = f(r, \theta, \zeta) \qquad \text{becomes} \qquad f(x, y_k - 1, s) = f(x, y_k, s)$$

- now must satisfy poloidal periodicity
  ◦ changing θ holding r, ζ constant changes both y_k and s

$$f(r, \theta + 1, \zeta) = f(r, \theta, \zeta) \qquad \text{becomes} \qquad f(x, y_k + q, s + 1) = f(x, y_k, s)$$

- now put each plane on its own coordinate system
  ◦ N drift planes: $s_{k+N} = s_k + 1$

$$f(x, y_k + q, s + 1) = f(x, y_k, s) \qquad \text{becomes} \qquad f(x, y_{k+N}, s_{k+N}) = f(x, y_k, s_k)$$

(details 2: parallel derivatives)

- special attention to unperturbed $\nabla_\parallel$

$$B\nabla_\parallel f = \frac{\partial f}{\partial s}$$

- finite difference across drift planes, each on its own coordinate system
  - (equidistant: $s_{k+1} - s_k = h_s$)

$$2h_s \left.\frac{\partial f}{\partial s}\right|_{s=s_k} = f(x, y_k, s_{k+1}) - f(x, y_k, s_{k-1})$$

$$= f(x, y_{k+1} - \Delta^+, s_{k+1}) - f(x, y_{k-1} - \Delta^-, s_{k-1})$$

- shifts come from coordinate definition $y_k = y - \alpha_k$

$$\Delta^\pm = \alpha_{k\pm 1} - \alpha_k$$

(details 3: brackets)

- transform using tensor rules (simplified form with $\Delta\alpha = 0$)
  - in general the only simplification is evaluation at $s = s_k$

$$\frac{\partial}{\partial r} = \frac{\partial x}{\partial r}\frac{\partial}{\partial x} + \frac{\partial y}{\partial r}\frac{\partial}{\partial y} = \frac{1}{a}\frac{\partial}{\partial x} + \frac{\partial q}{\partial r}(s - s_k)\frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial \theta} = \frac{\partial y}{\partial \theta}\frac{\partial}{\partial y} + \frac{\partial s}{\partial \theta}\frac{\partial}{\partial s} = q\frac{\partial}{\partial y} + \frac{\partial}{\partial s}$$

- fluxtube ordering: $k_\parallel \ll k_\perp$ implies $\partial/\partial s \ll \partial/\partial x$ or $\partial/\partial y$

$$\frac{1}{r} \quad \text{becomes} \quad \frac{1}{a} \quad \text{hence} \qquad [\phi, f] = \frac{c}{B_0 a^2}\left(\frac{\partial \phi}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial \phi}{\partial y}\frac{\partial f}{\partial x}\right)$$

- caveat on the curvature: $\partial/\partial y = 0$ for $\log B^2$ hence keep $\partial/\partial s$ for it

$$-[\log B^2, f] \to \mathcal{K}(f) = \frac{c}{B_0 a}\frac{2}{R_0}\left[(\cos s + g_k^{xy}\sin s)\frac{\partial f}{\partial y} + \sin s\frac{\partial f}{\partial x}\right]$$

(details 4: Arakawa's discretisation for brackets)

- form various versions of "Jacobian" operation (straight, rotational, diagonal)
  - evaluated at grid node 00 with + or − neighbours in xy plane

$$J^{++} = \frac{1}{4h^2}[(\phi_{+0} - \phi_{-0})(f_{0+} - f_{0-}) - (\phi_{0+} - \phi_{0-})(f_{+0} - f_{-0})]$$

$$J^{+\times} = \frac{1}{4h^2}[\phi_{+0}(f_{++} - f_{+-}) - \phi_{-0}(f_{-+} - f_{--}) - \phi_{0+}(f_{++} - f_{-+}) + \phi_{0-}(f_{+-} - f_{--})]$$

$$J^{\times+} = \frac{1}{4h^2}[\phi_{++}(f_{0+} - f_{+0}) - \phi_{--}(f_{-0} - f_{0-}) - \phi_{-+}(f_{0+} - f_{-0}) + \phi_{+-}(f_{+0} - f_{0-})]$$

$$J^{\times\times} = \frac{1}{8h^2}[(\phi_{++} - \phi_{--})(f_{-+} - f_{+-}) - (\phi_{-+} - \phi_{+-})(f_{++} - f_{--})]$$

- demand antisymmetry of bracket, conservation of energy and enstrophy, find

$$[\phi, f]_{xy} = \frac{1}{3}\left(J^{++} + J^{+\times} + J^{\times+}\right)$$

(details 5: Karniadakis's time step)

- a variant on the Adams/Bashforth theme, expand both sides 3 timesteps deep
  - this recovers all mixed terms in time/space Taylor expansion

$$\frac{\partial f}{\partial t} = S \qquad \text{with} \qquad \sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{\Delta t} = \sum_{j=1}^{3} \beta_j \, S_j$$

- coefficients for order 3:

$$\alpha_{1,2,3} = \quad 3 \quad -3/2 \quad 1/3 \qquad \beta_{1,2,3} = \quad 3 \quad -3 \quad 1$$

- incorporation of an implicit dissipation piece L is straightforward
  - watch out for the factor of 6/11 (inverse sum over the α_j)
  - NB: always avoid implicit techniques with wave dynamics

$$\sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{\Delta t} + L(f_0) = \sum_{j=1}^{3} \beta_j \, S_j$$

Basic Situation in the Tokamak Edge

- edge time scales for electrons
  - collisions ν_e
  - thermal transit V_e/qR
  - Alfvén transit v_A/qR
  - turbulence 10^{-2} to 1 times c_s/L_T

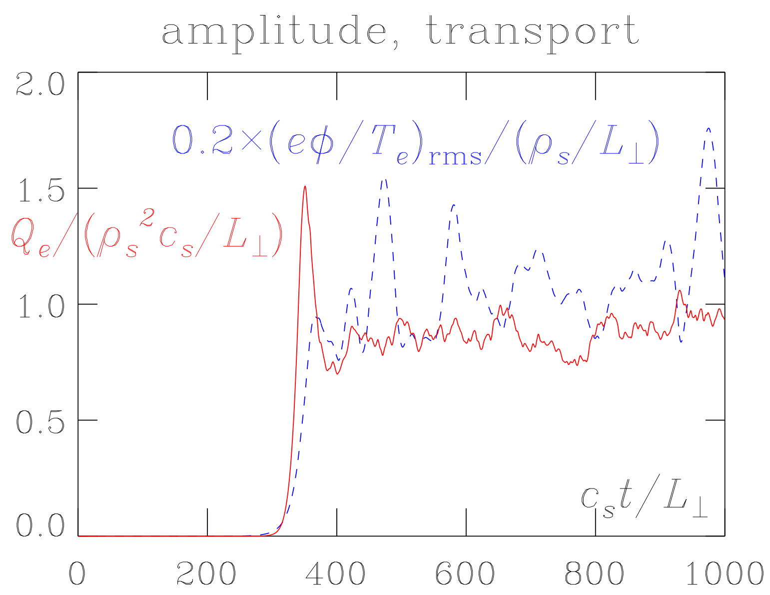
- edge time scales for ions
  - collisions ν_i
  - thermal transit c_s/qR

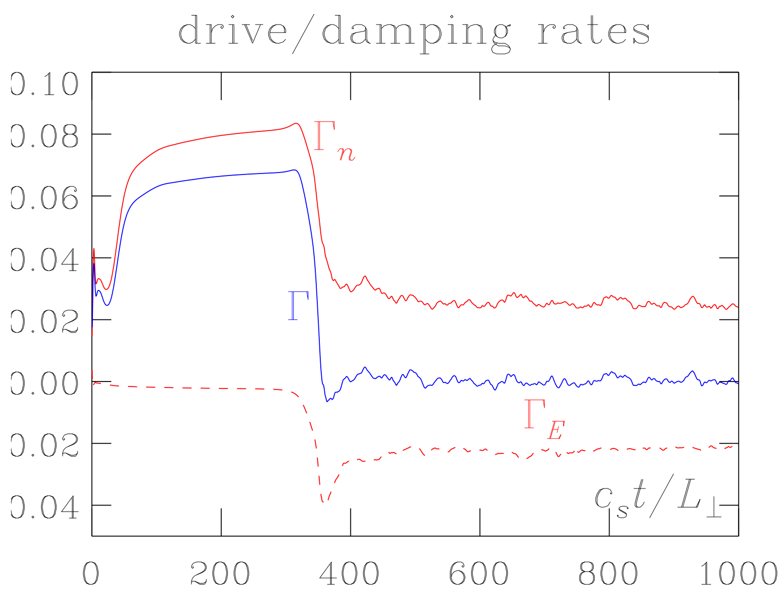
electron time scales comparable to turbulence ion time scales *much slower*

$$\hat{\beta} = \left(\frac{c_s/L_\perp}{v_A/qR}\right)^2 \qquad \hat{\mu} = \left(\frac{c_s/L_\perp}{V_e/qR}\right)^2 \qquad C = \frac{0.51\nu_e}{c_s/L_\perp}\hat{\mu} \qquad \text{all} > 1$$

Nonlinear Saturation

basic feature of any instability –– transition to turbulence





linear drive (n) ––> linear growth

moment of saturation –– growth rate (T) drops to zero

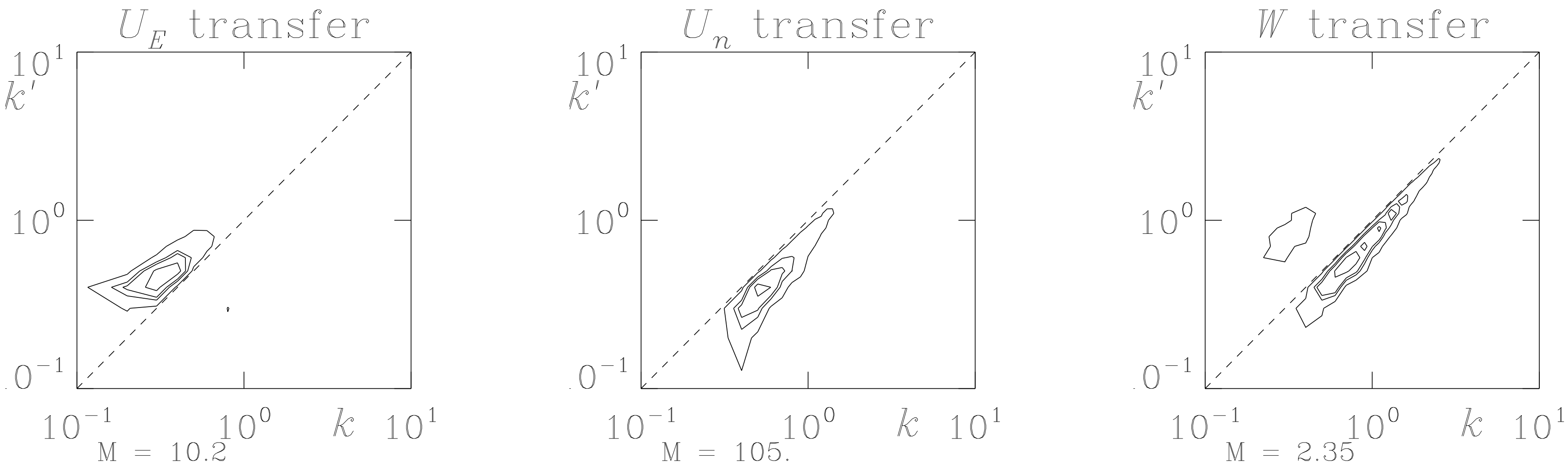
saturation maintained –– nonlinear transfer to subgrid scale dissipation (E)

transport (Q) overshoots, finds saturated balance

(B Scott Phys Plasmas 6/2005)

Nonlinear Cascade in Turbulence

basic statistical character of three wave energy transfer



transfer between wavenumber magnitudes –– from k' to k

all activity near the k' = k line ––> cascade character

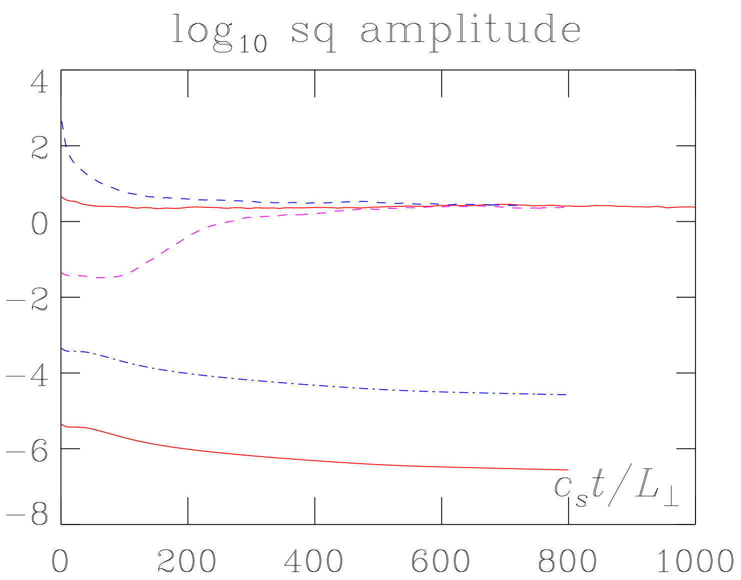
ExB energy is inverse, while other quantities are direct (to higher k)

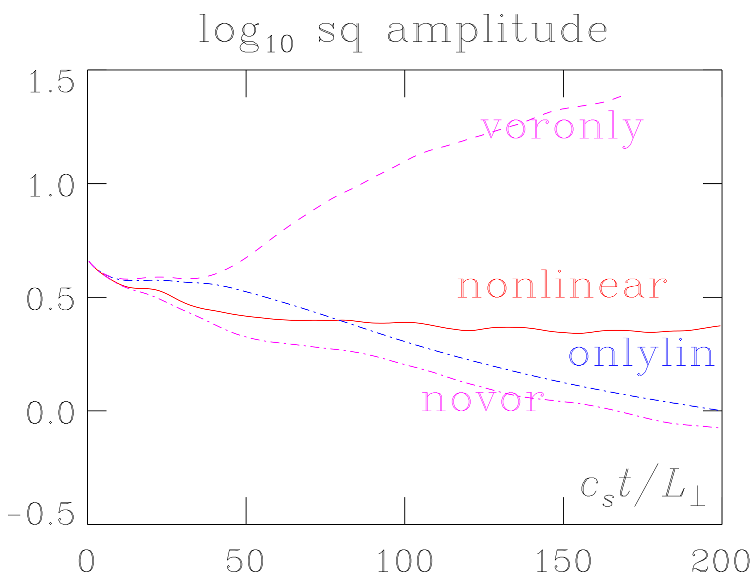
dominant transfer is through the thermal free energy (n), others also active

(S Camargo et al Phys Plasmas 1995, 1996)

Nonlinear Instability

basic feature of drift wave turbulence (edge turbulence test case)





amplitude threshold −−> linear stability

vorticity nonlinearity −−> damped eigenmodes destabilise each other

role of pressure advection nonlinearity −−> saturation

edge turbulence −−> washes out microinstabilities in toroidal magnetic field

(B Scott Phys Rev Lett 1990, Phys Fluids B 1992, New J Phys 2002)

Energy Transfer

part of energy theorem governed by vorticity equation

$$-\phi_{-k}\left(\dot{\Omega} + v_E \cdot \nabla \Omega + \mathrm{FLR} = \nabla_\parallel J_\parallel + \nabla \cdot \frac{c}{B^2} B \times \nabla p\right)_k$$

Fourier mode k

vorticity $\Omega = (n_e - n_i)\, e$

currents: polarisation parallel diamagnetic

free energy: source in pressure equation, transfer in to vorticity equation

pathways: over parallel dynamics or toroidal compression

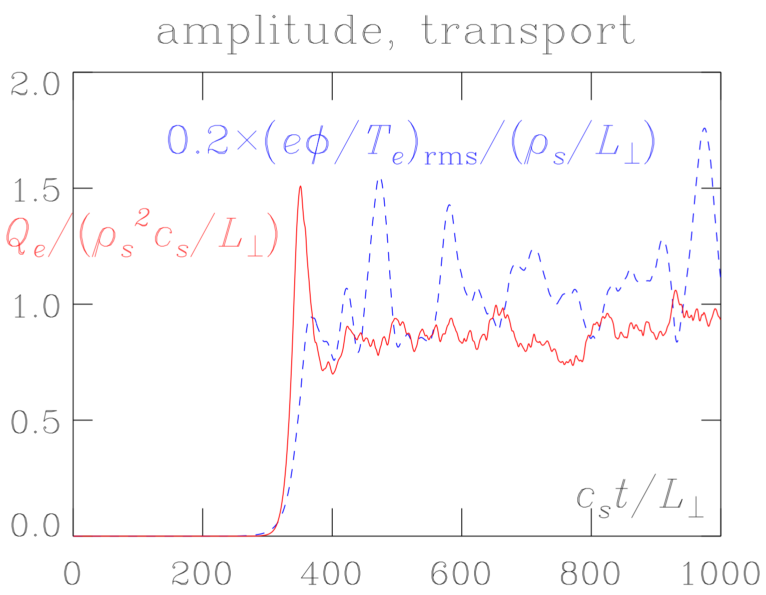
between modes within ExB energy –– nonlinear advection

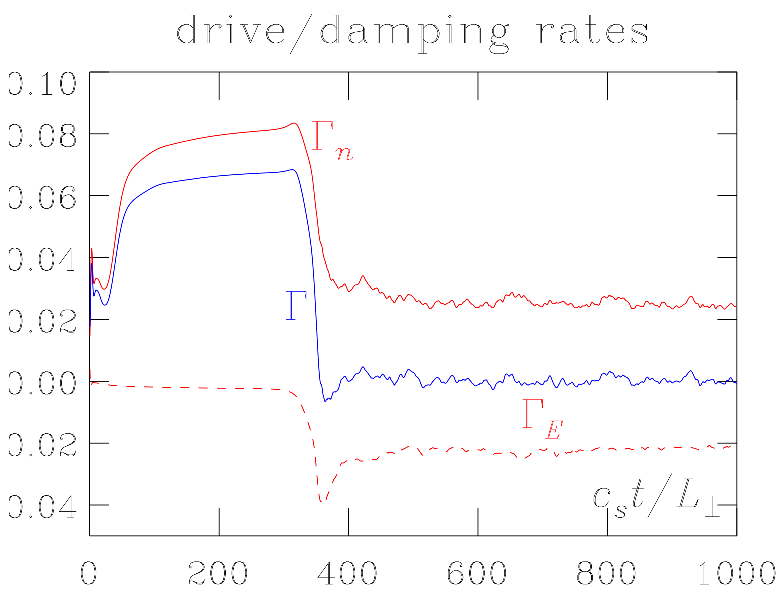
direct, in–context measurement of physical mechanism supporting turbulence

(B Scott Phys Plasmas 2000)

Nonlinear Saturation

basic feature of any instability –– transition to turbulence





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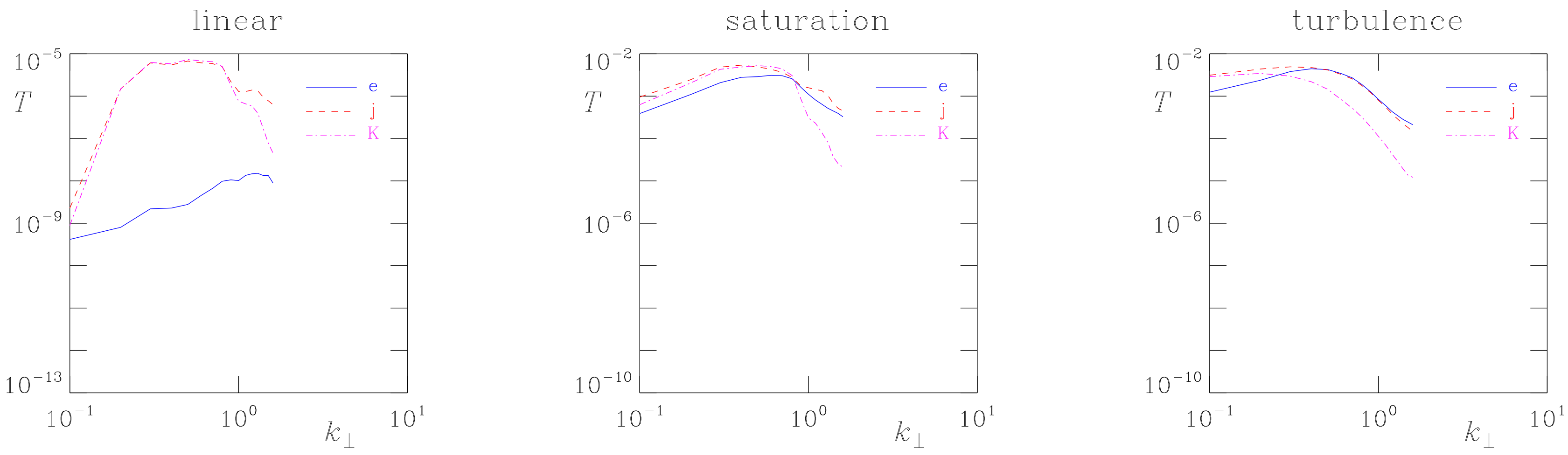
(B Scott Phys Plasmas 6/2005)

Vorticity Energetics –– Transition to Turbulence

turbulence imposes its own mode structure on dynamics



linear interchange mode –– balance between diamagnetic/parallel currents

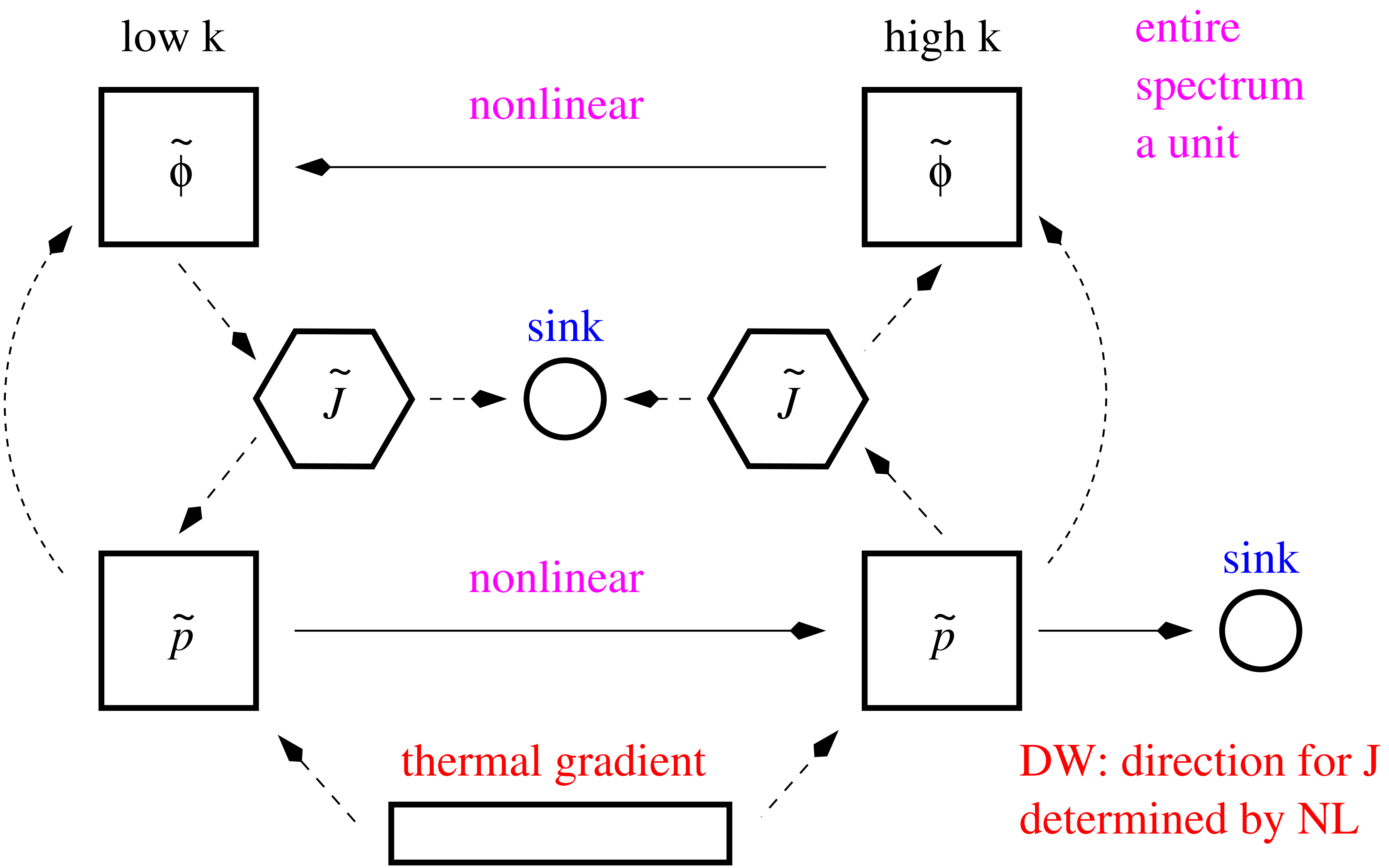
turbulence –– emergence of nonlinear ExB vorticity advection

developed turbulence –– balance between polarisation/parallel currents

basic mechanism supporting eddies in turbulence differs from linear instability

(B Scott Plasma Phys Contr Fusion 2003)

Energy Transfer: electromagnetic turbulence



(B Scott Phys Fluids B 1992, Plasma Phys Contr Fusion 1997)

(S Camargo et al Phys Plasmas 1995 and 1996) blank page to fool ps2pdf

electromagnetic cases — notes

- nominal value of beta

$$\hat{\beta} = \frac{4\pi p_e}{B^2} \left(\frac{qR}{L_\perp} \right)^2 = 1.75$$

- introduction of "flutter" effects

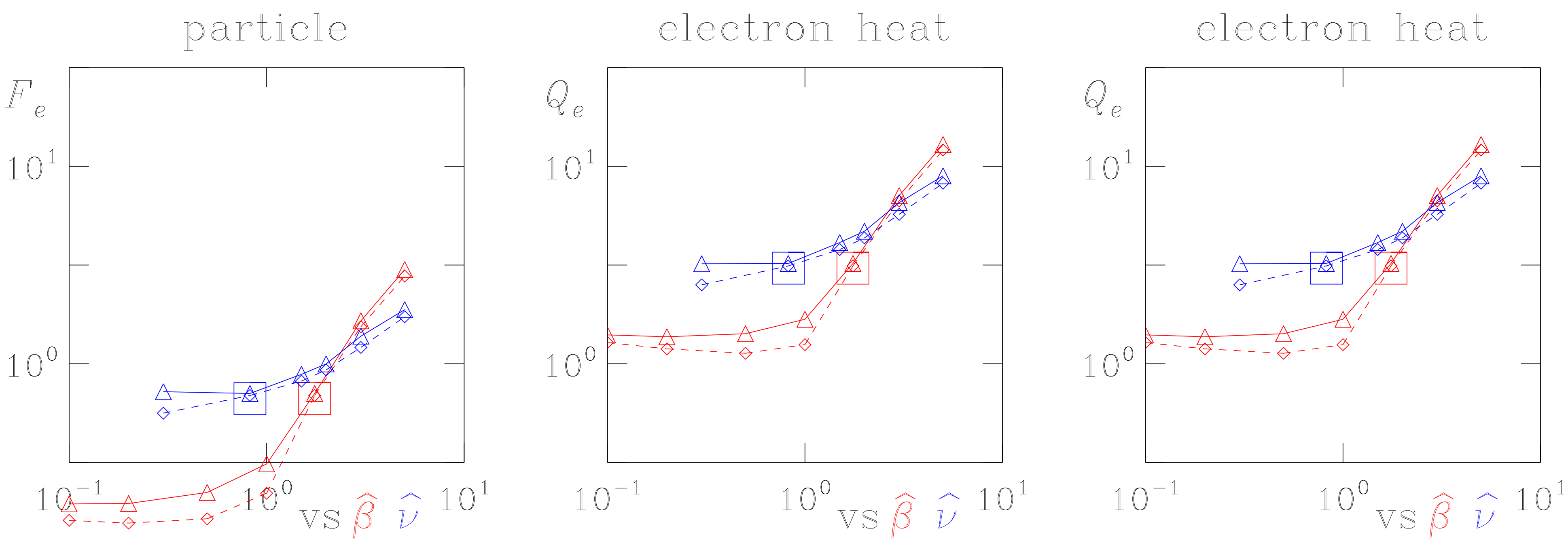
$$\nabla_\parallel = b^s \frac{\partial}{\partial s} - \hat{\beta}[A_\parallel, \,] \qquad\qquad -\nabla_\perp^2 A_\parallel = J_\parallel \leftrightarrow \nabla_\parallel (p_e - \phi)$$

- as $\hat{\beta}$ rises from zero, transport is relatively insensitive, then rises
  - ○ linearly weak interchange/ballooning modes are available at low-k_y
  - ○ these are driven via nonlinear cascade, through turbulence vorticity

- at high-k_y turbulence vorticity overcomes linear instabilities
  - ○ rough rule of thumb: $\omega_* > \gamma_I$ (diamag freq > MHD interchange growth rate)
  - ○ edge conditions: $k_y \rho_s > 2L_\perp/R$ since $\rho_s/L_\perp \gtrsim L_\perp/R$

Basic Edge Transport Scaling

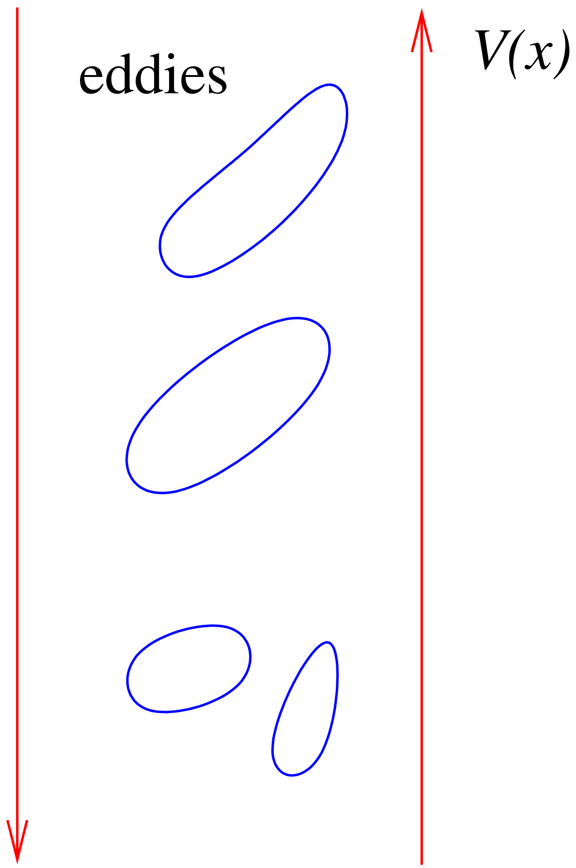
mid-size tokamak L-mode cases, local geometry, $64 \times 256\rho_s \times 2\pi qR$ domain

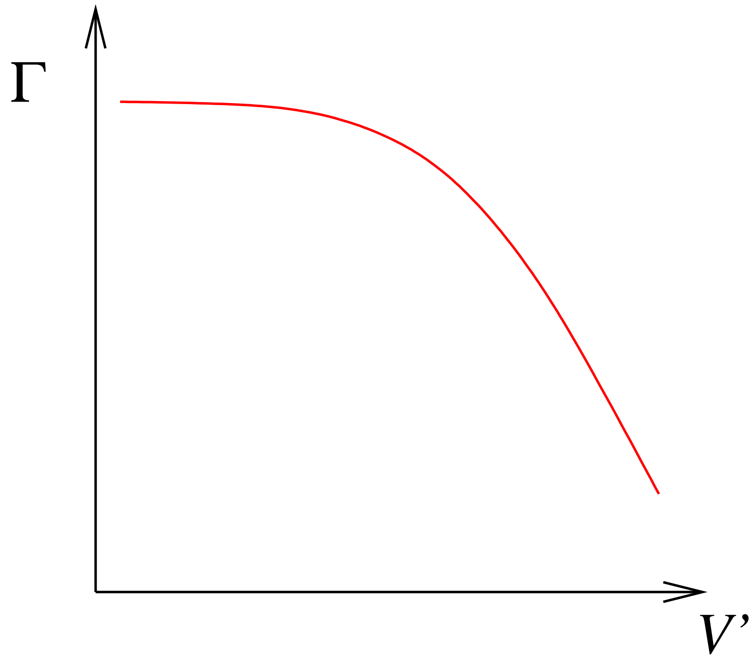


- synergy: all three transport channels vary together (squares: nominal case)
- beta turnup due to long wavelength nonlinear transfer (dashes: $2\times$ resolution)
- in both cases sensitivity is due to nonadiabatic electrons

Suppression of Turbulence by Flows

(Biglari Diamond Terry, Phys Fl B 1991)



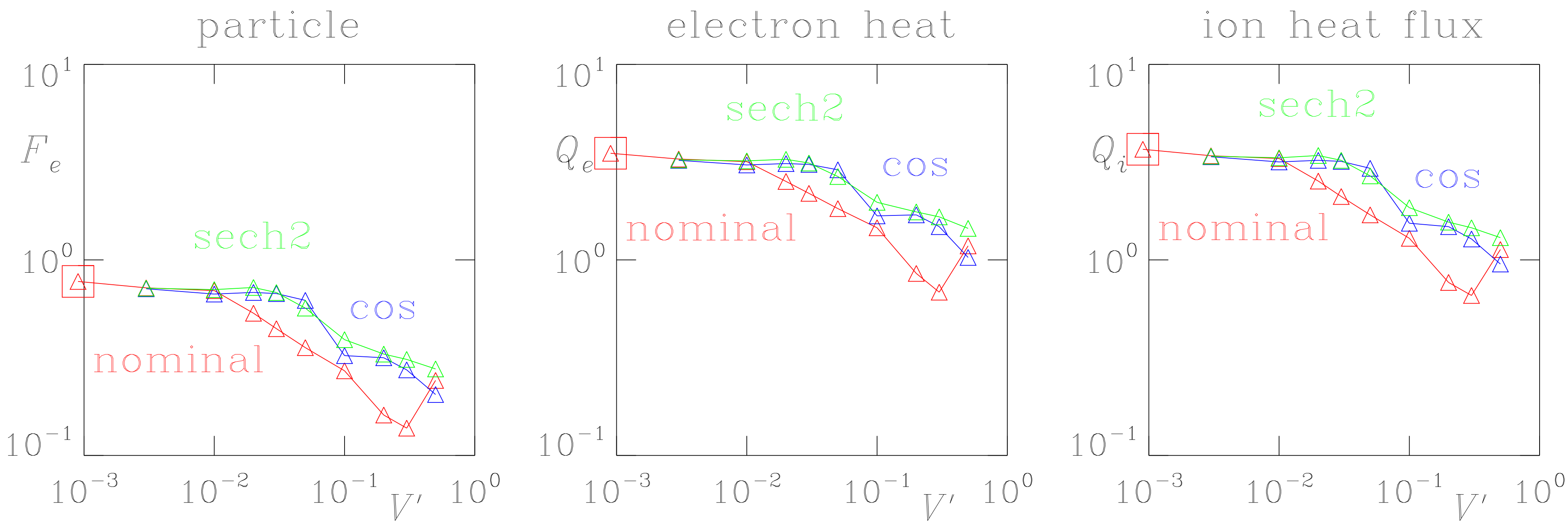


eddies tilted into energy–losing relationship to flow vorticity

––> same process as in self generation

Sensitivity to Externally Imposed ExB Shear

standard L-mode cases with ITG gradients: $L_T = 0.5\, L_n$



- squares give value at zero shear
  - red/blue/green lines give constant/cos/sech2 profiles for applied vorticity
- rolloff is slow, no steeper than $Q \sim (V')^{-1}$
- max suppression is only about a factor of 4

Main Points — Transport Scaling

- trends follow nonlinear, not linear, physics

- effects of ion grad-T must be kept...
  - accounts for nonlinearly driven longer wavelengths
  - prevents cutoff of transport towards higher T and grad-T

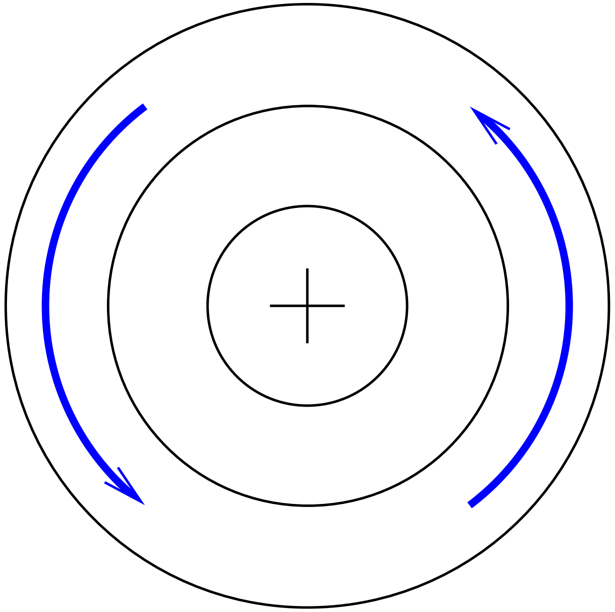
- trend with either grad-T or beta always monotonically upward

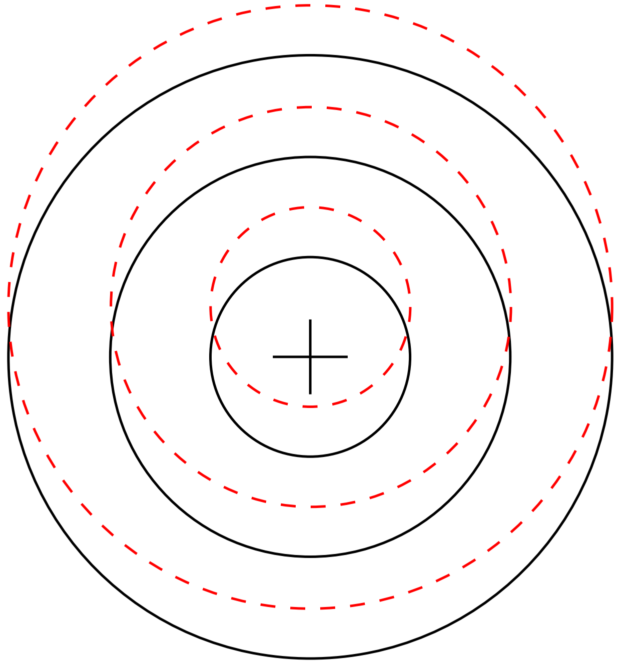
- shear flow suppression too weak to overcome beta scaling

> no L-to-H transition in fully developed turbulence in local models

Zonal Flow, Toroidal Compression

(Winsor et al Phys Fl 1968, Hahm et al Plasma Phys Contr Fusion 2002, 2004)





zonal flow

$<\phi>$

compression at top

divergence at bottom

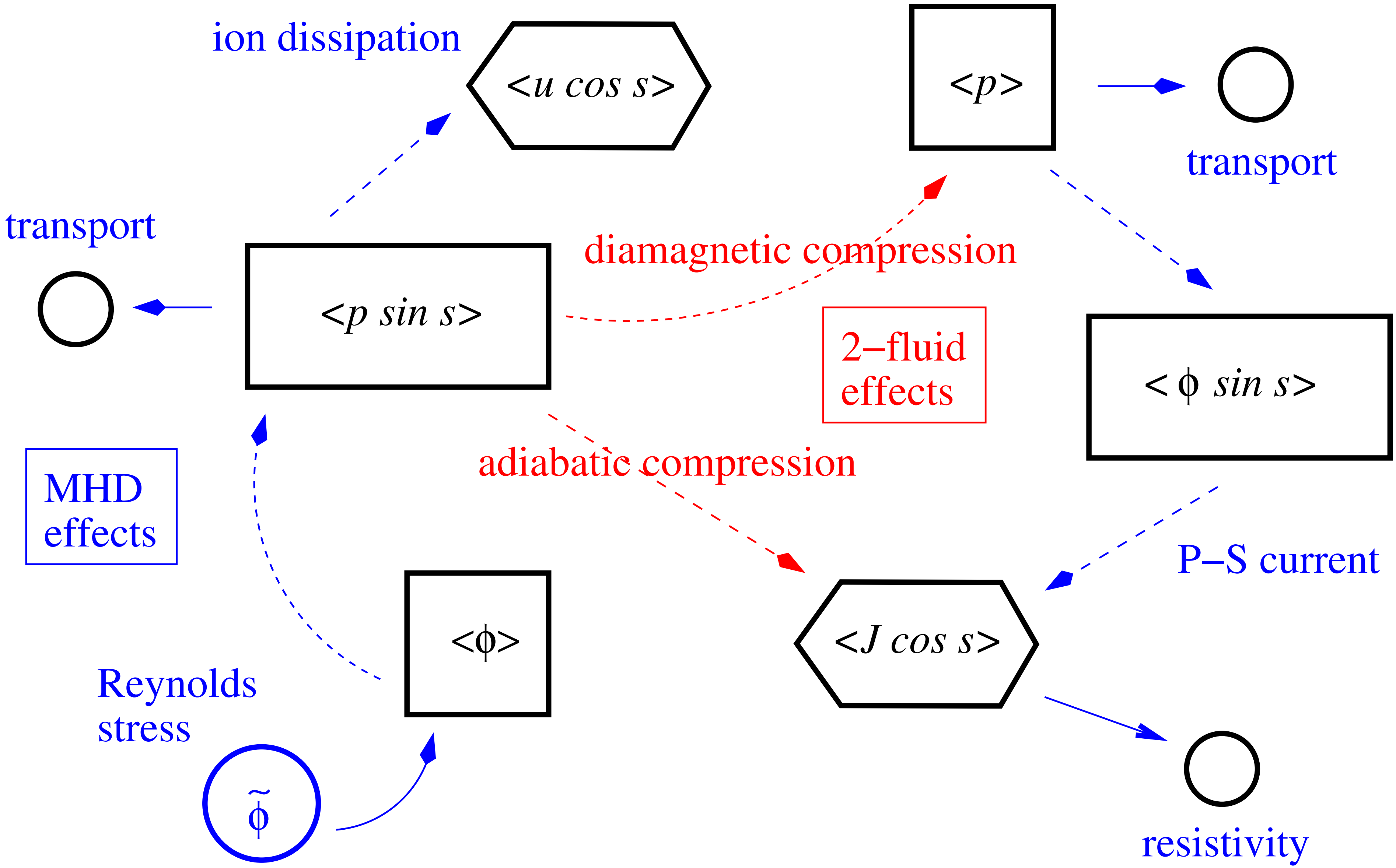
pressure sideband

$<p \sin\theta>$

zonal flow exchanges conservatively with pressure sideband

--> transfer pathway, equipartition

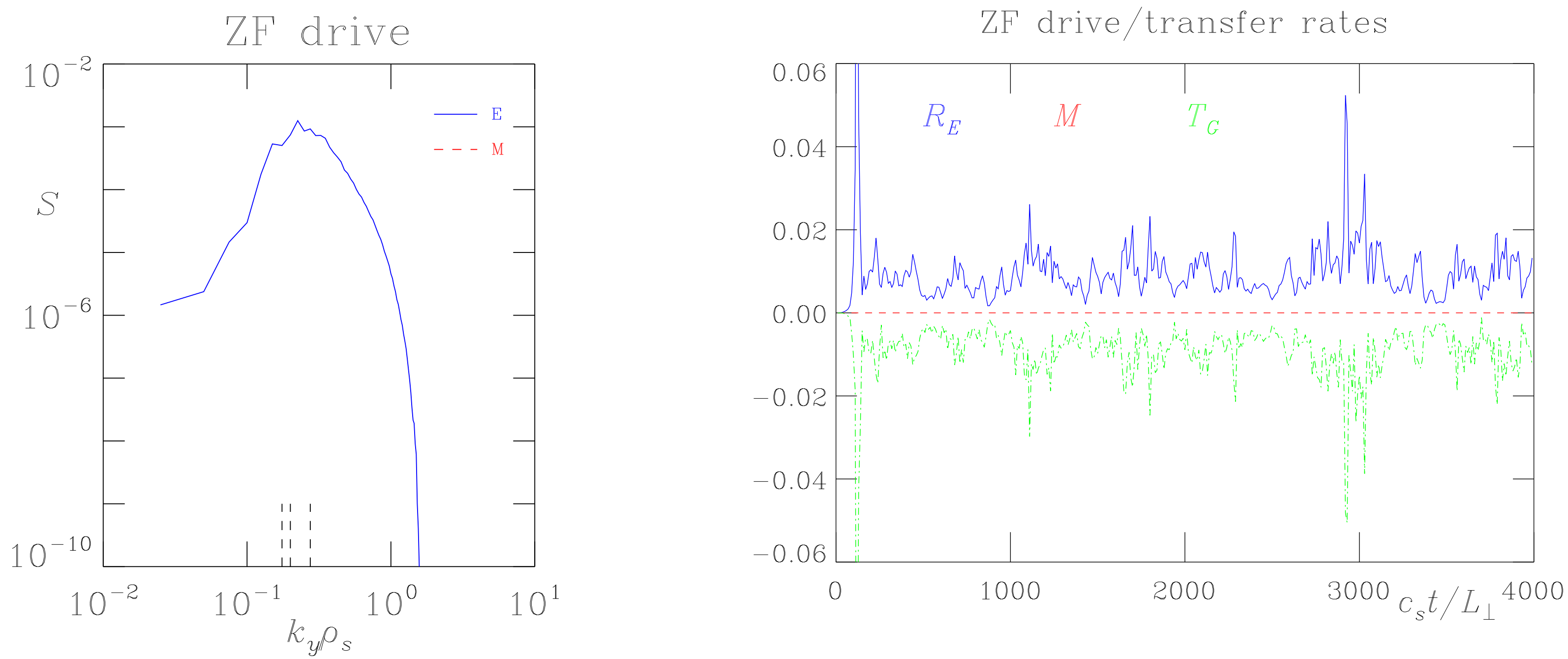
Energy Transfer: flows and currents



(B Scott Phys Lett A 2003, New J Phys 2005)

Coupling to Zonal Flows

turbulence regulated by flows, regulated by toroidal compression



eddy Reynolds stress −−> energy transfer from turbulence to flows

turbulence moderately weakened but not suppressed

toroidal compression −−> energy loss channel to pressure, turbulence

entire system in self regulated statistical equilibrium (turb, flows, mag eq)

(B Scott Phys Lett A 2003, New J Phys 2005)

Including the Self-consistent Profile Evolution

- allow the turbulence advection (mixing) to evolve the profile
  ◦ here, "profile" is the same as "zonal component"

- now the profile is part of the dependent variable
  ◦ it is acted upon by magnetic curvature (toroidal compressibility of drifts)

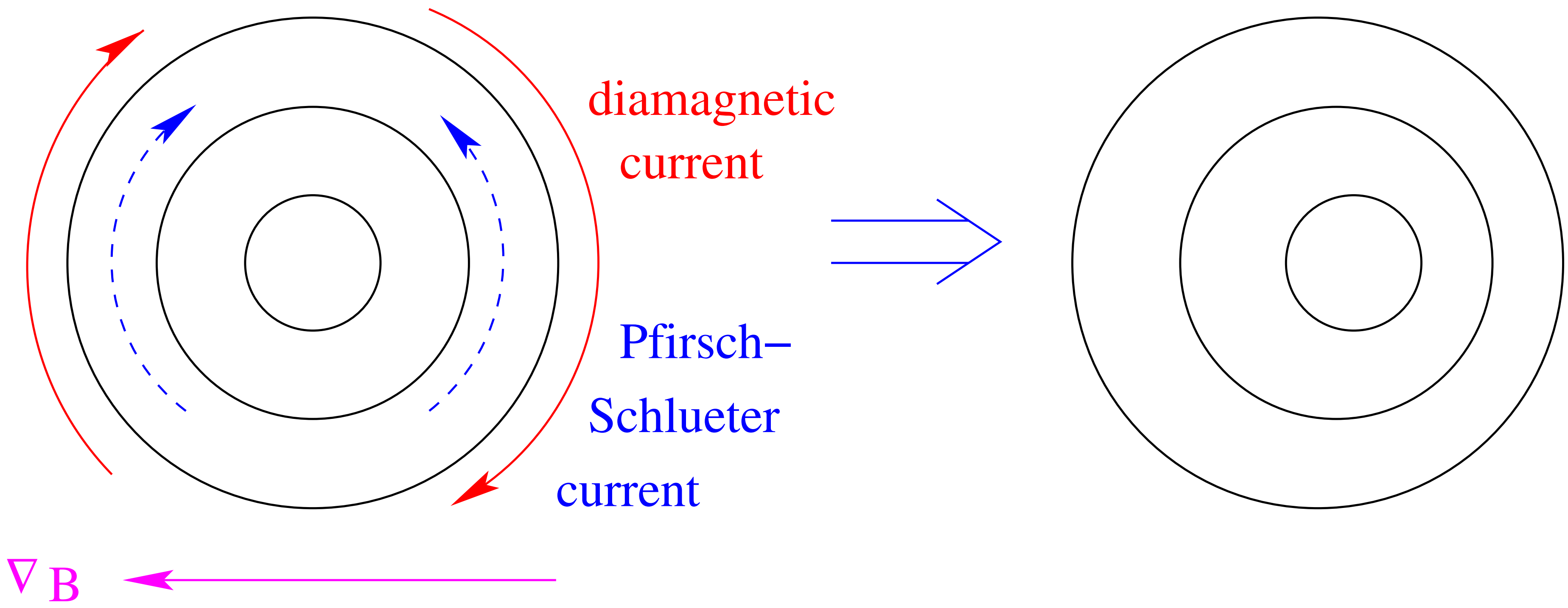
- hence the "neoclassical equilibrium" is necessarily a part of the evolution
  ◦ flow balance: zonal flows, geodesic curvature coupling,
  nonlinear transfer to turbulence → zonal flow saturation
  ◦ current balance: Pfirsch-Schlüter current, Shafranov shift

$$\frac{\partial}{\partial x}\left\langle A_\parallel \right\rangle \rightarrow \delta\frac{1}{q} \qquad\qquad \left\langle A_\parallel \cos s \right\rangle \rightarrow \text{Shafranov shift}$$

all of the above must now be carried self consistently

Incorporation of Magnetic Equilibrium

toroidal equilibration current <--> Shafranov shift



P–S current equilibrates toroidal diamagnetic compression

Ampere's Law --> ''Pfirsch–Schlueter magnetic field'' --> toroidal shift

current stays in moment variables, magnetic field in coordinate metric

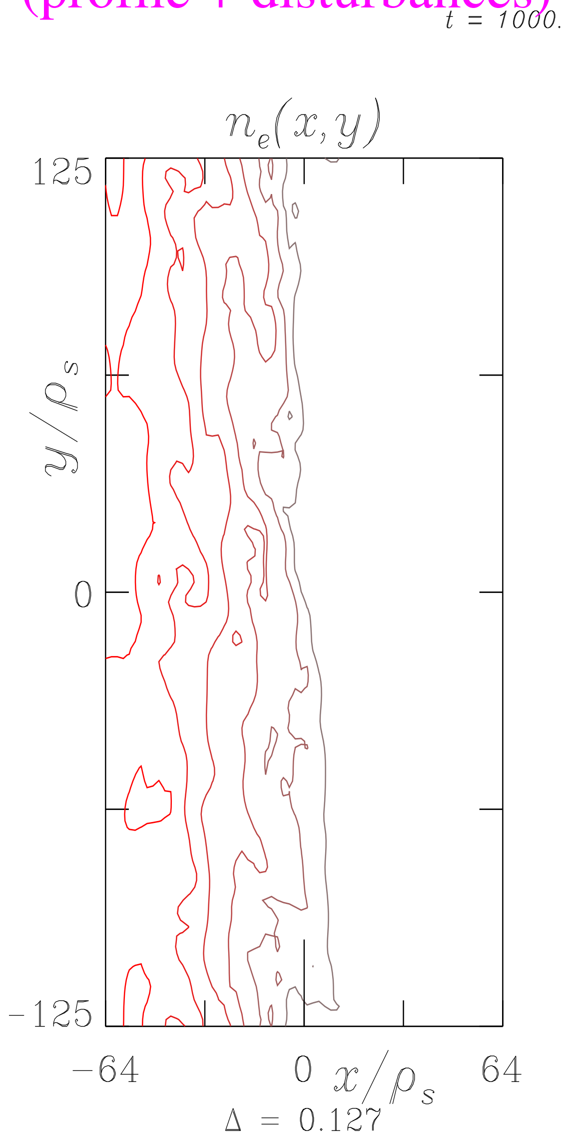
Global Electromagnetic Gyrofluid (GEM):

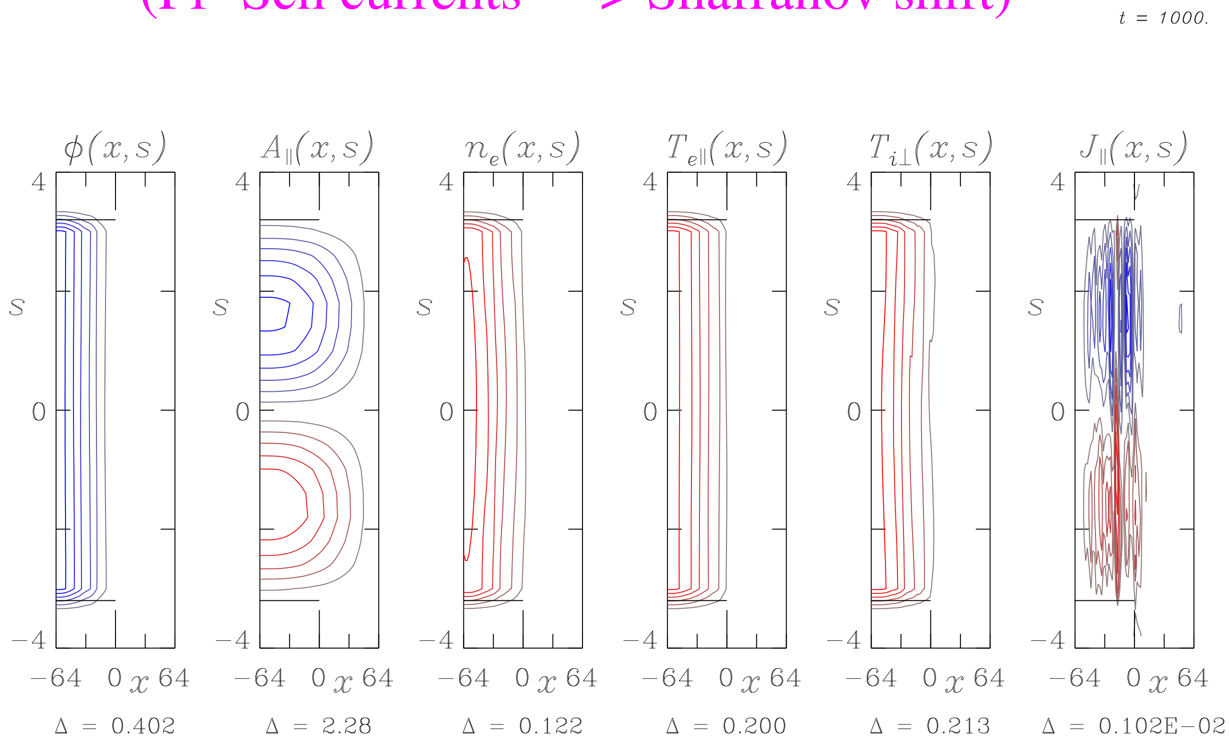
turbulence and transport

(profile + disturbances)

self consistent magn eq, geometry

(Pf–Sch currents ––> Shafranov shift)





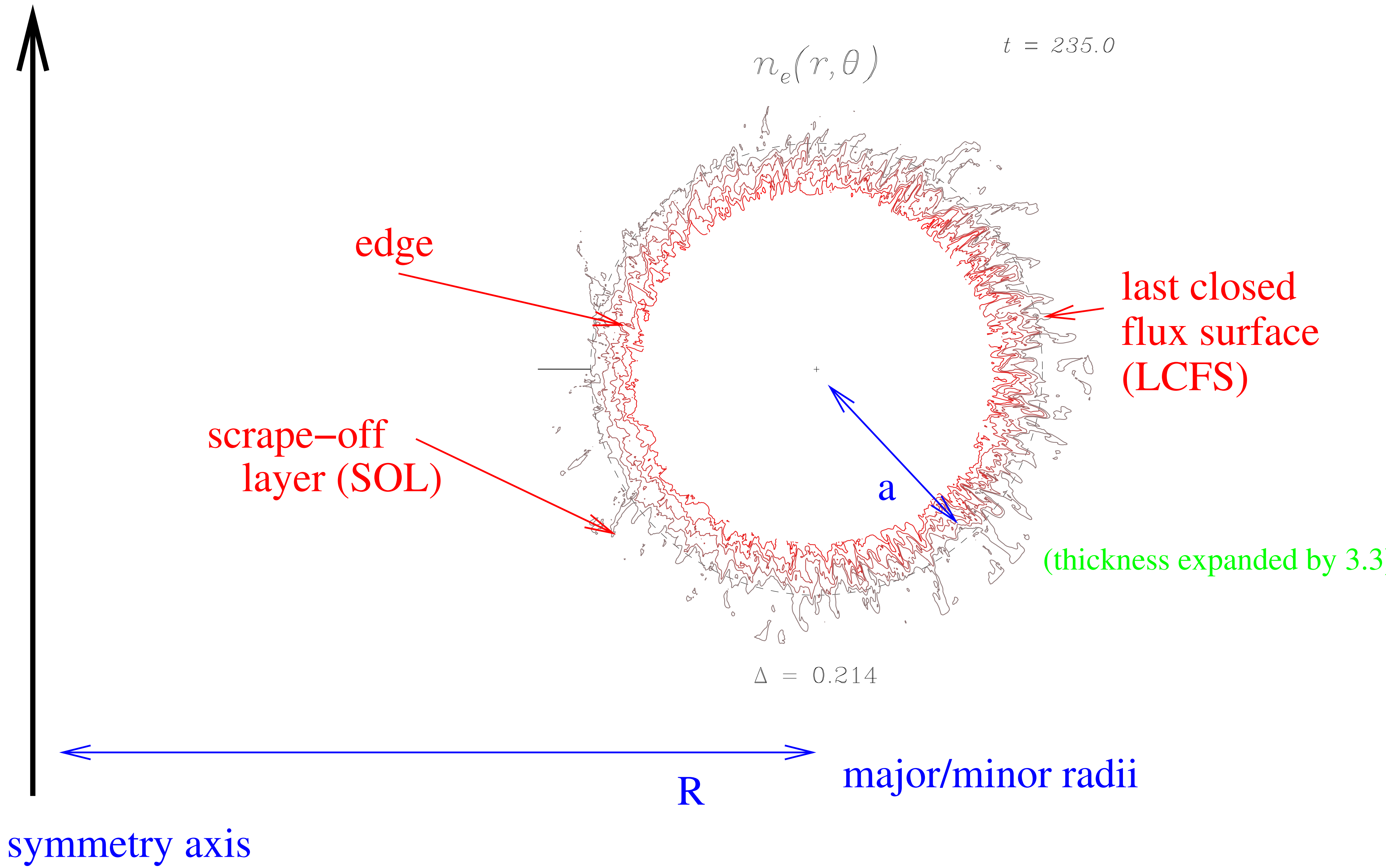
L–Mode Base Case (ASDEX Upgrade generic)

correct mass ratio, gyroradius

closed/open flux surfaces, separatrix topology

(B Scott Contrib Plasma Phys 2006)

A Typical Burst Event



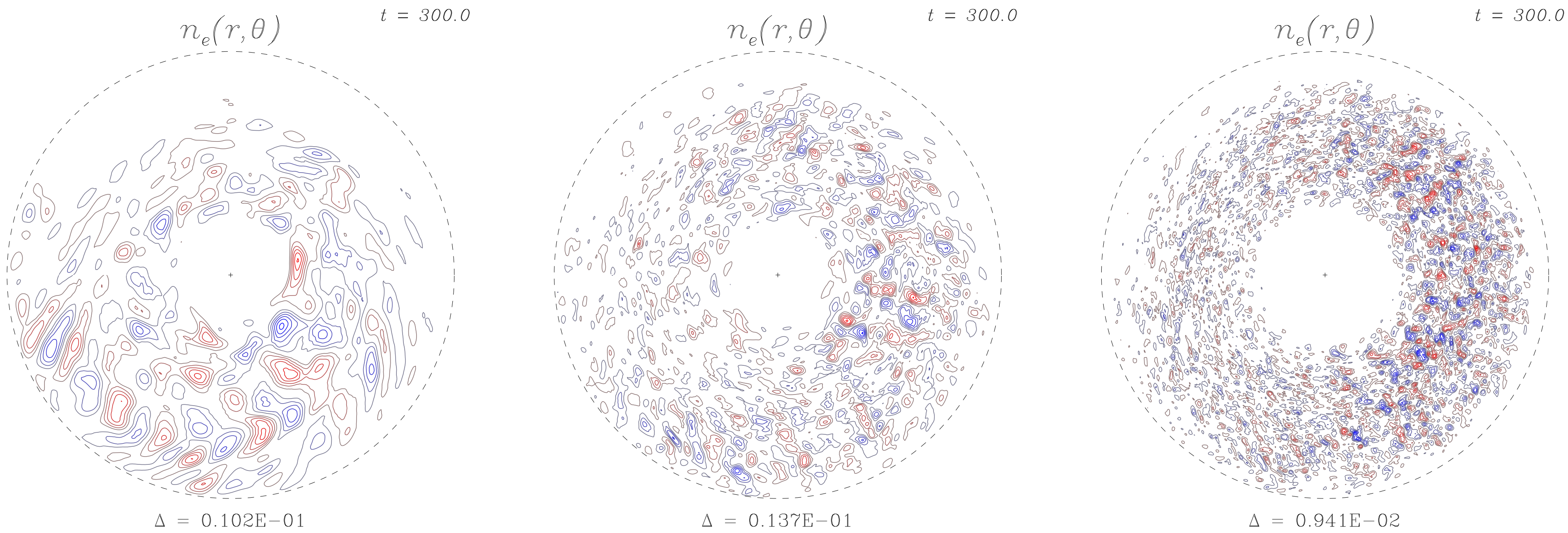
Scale Separation

- turbulence vorticity scales with $c_s/L_\perp$, velocity with $c_s(\rho_s/L_\perp)$
- transport flux scales with $c_s(\rho_s/L_\perp)^2$, diffusivity with $c_s\rho_s^2/L_\perp$
- this is called "gyro-Bohm" and arises in general from $\rho_s \ll L_\perp$
- edge layer confinement time scales with $L_\perp^2/D$ or $(L_\perp/c_s) \times (L_\perp/\rho_s)^2$
- for edge (not SOL) turbulence this is about 1 msec with $L_\perp/\rho_s \gtrsim 50$

it is vital to get this correct in a computation
since the turbulence/equilibrium crosstalk depends on it

Scale Separation Look and Feel

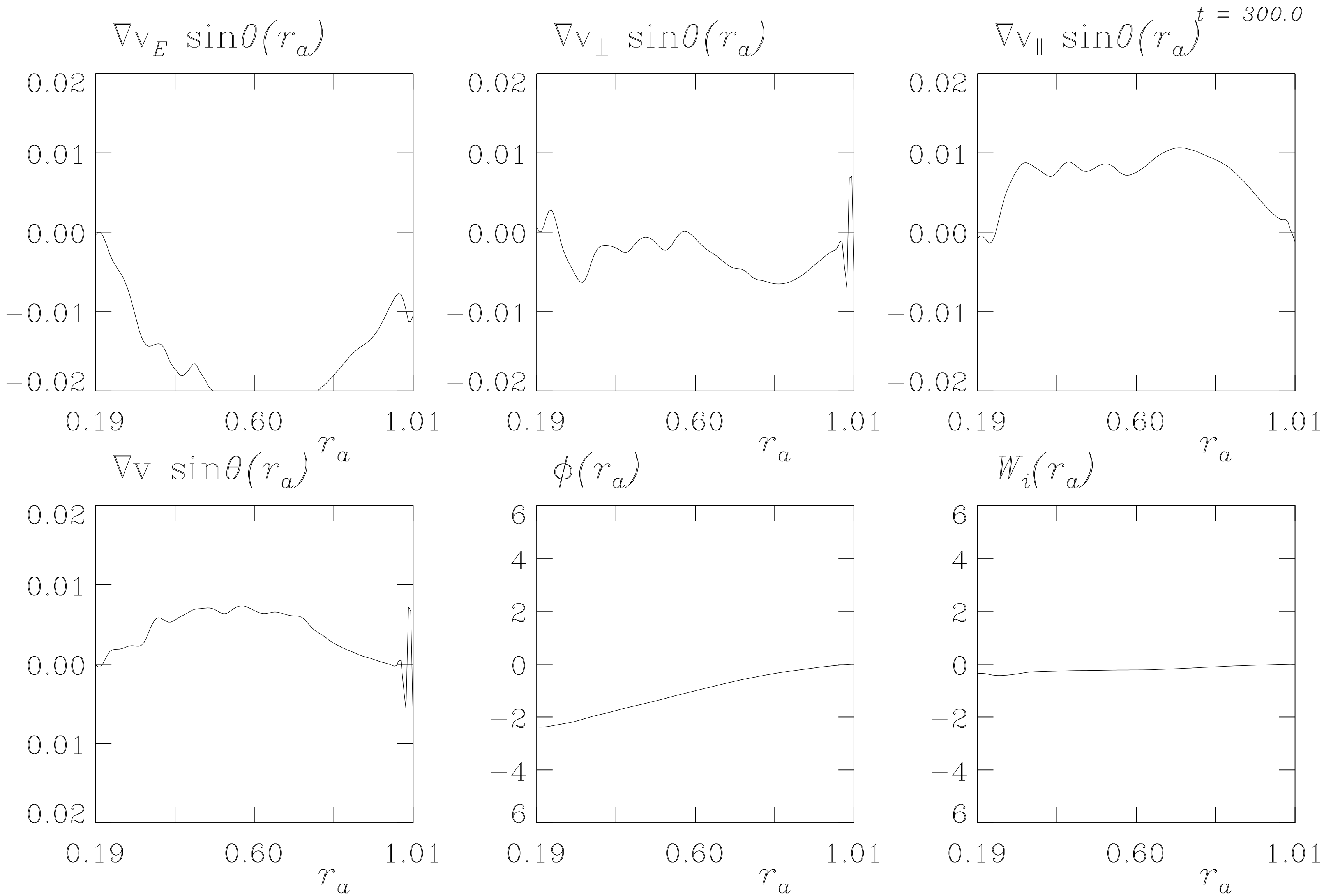
electromagnetic core cases with a/ρ_s of 50, 100, and 200, non-axisymmetric part



- if you can see the eddies on a global plot they're too large!
- in the edge you have $L_\perp/\rho_s < 100$ but $2\pi a/q > 10^3\rho_s$

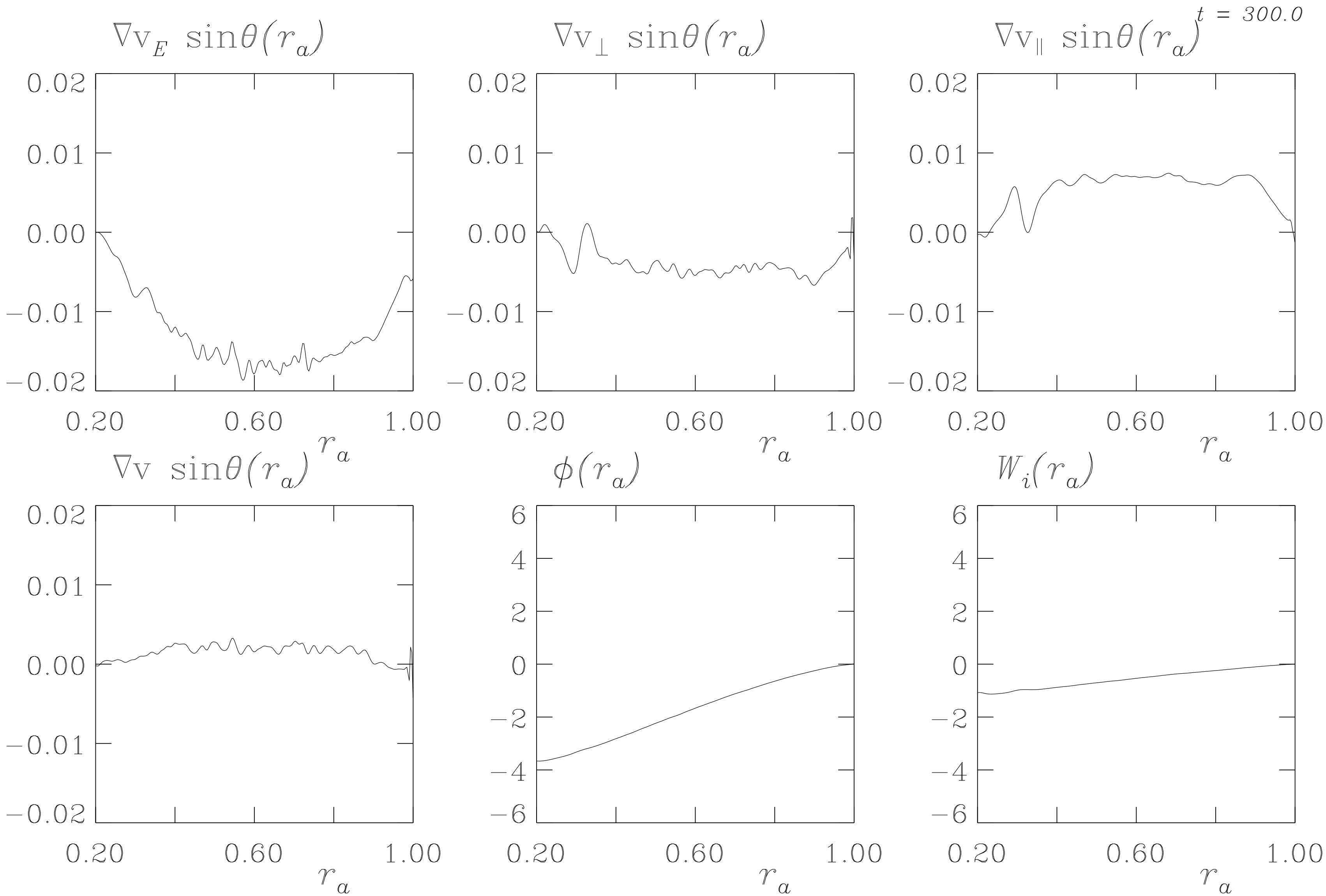
Ion Flow Sideband Divergences — Small Case

- flow divergence pieces do not balance



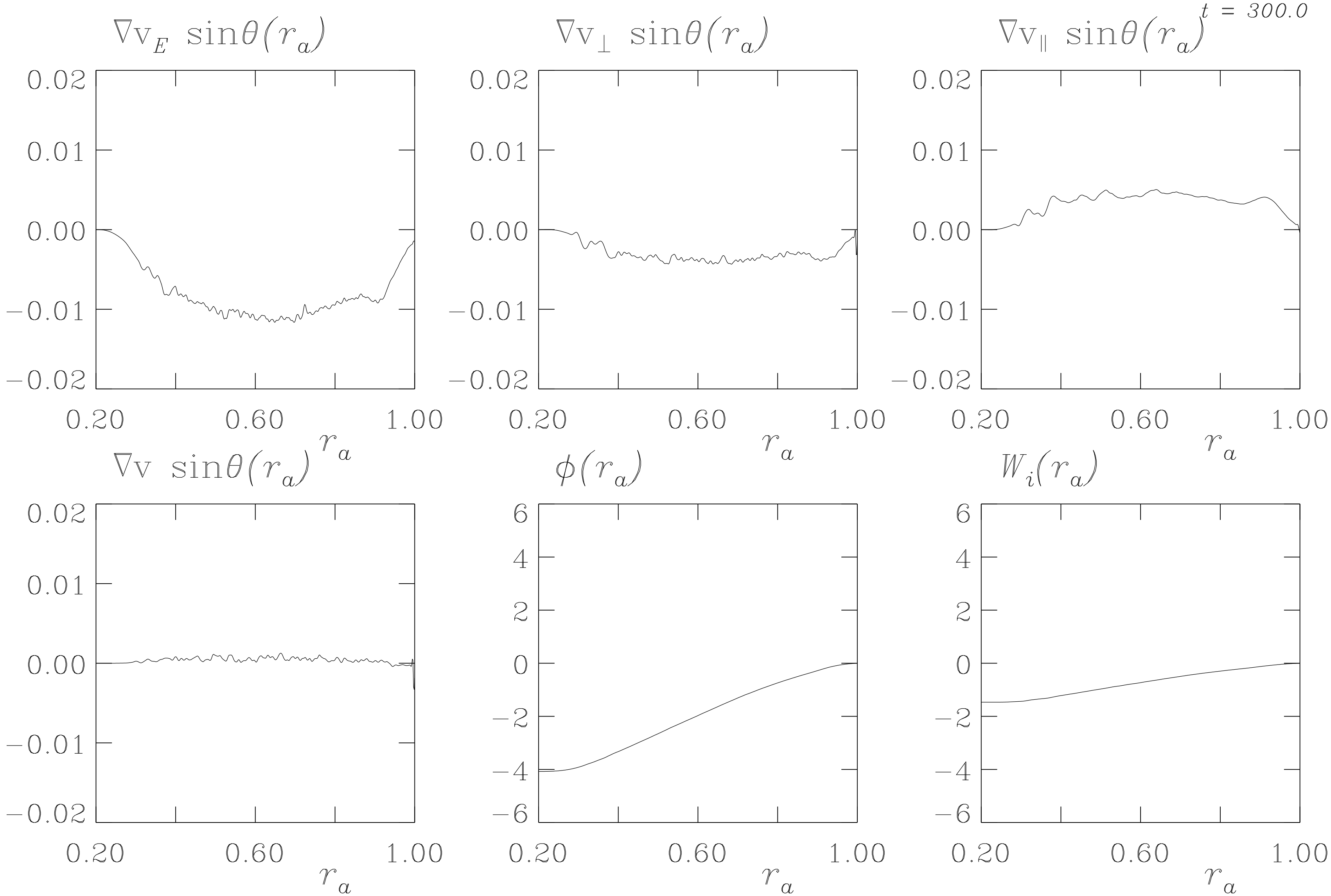
Ion Flow Sideband Divergences — Medium Case

- flow divergence pieces almost balance



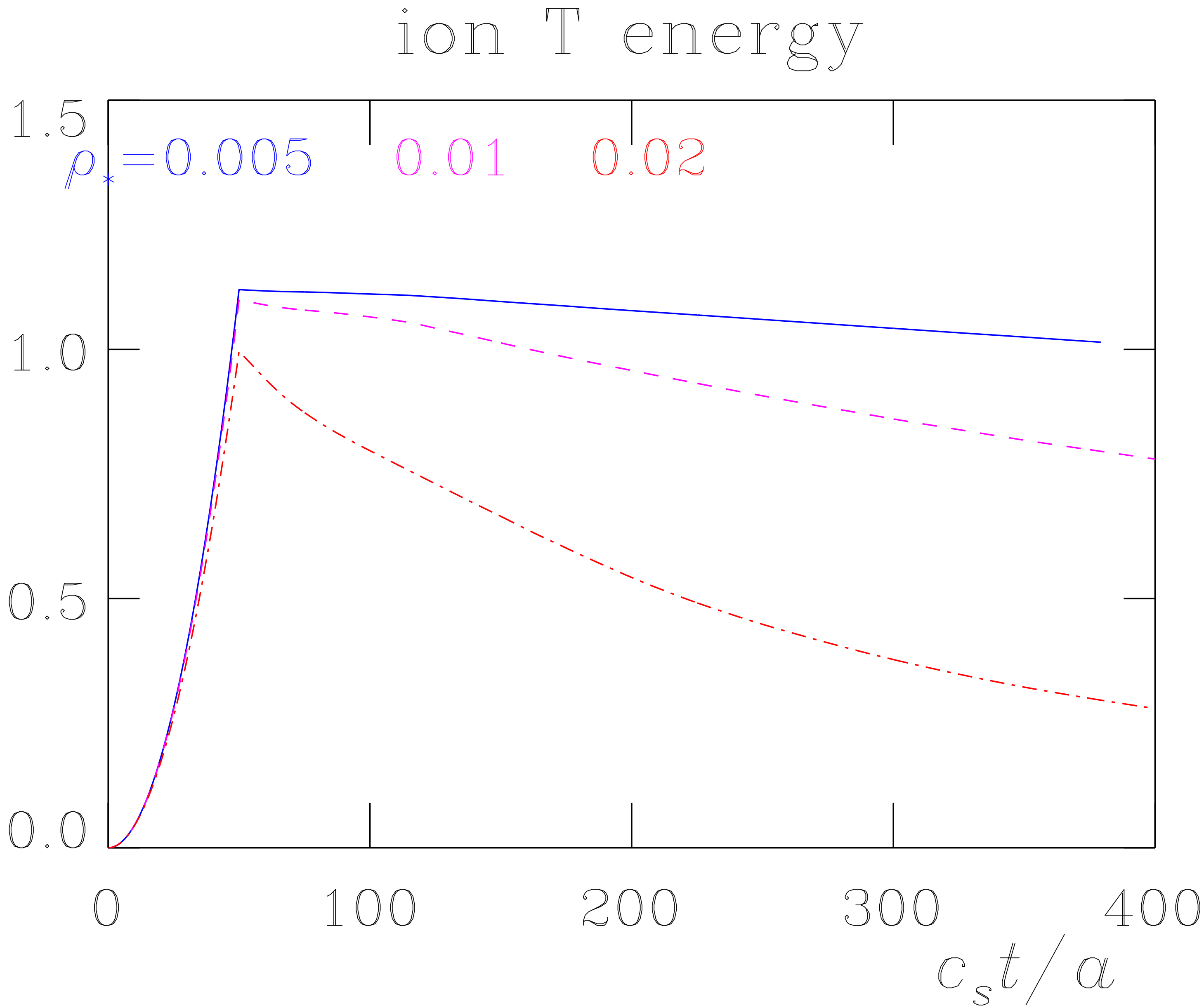
Ion Flow Sideband Divergences — Nominal Case

- flow divergence pieces balance closely



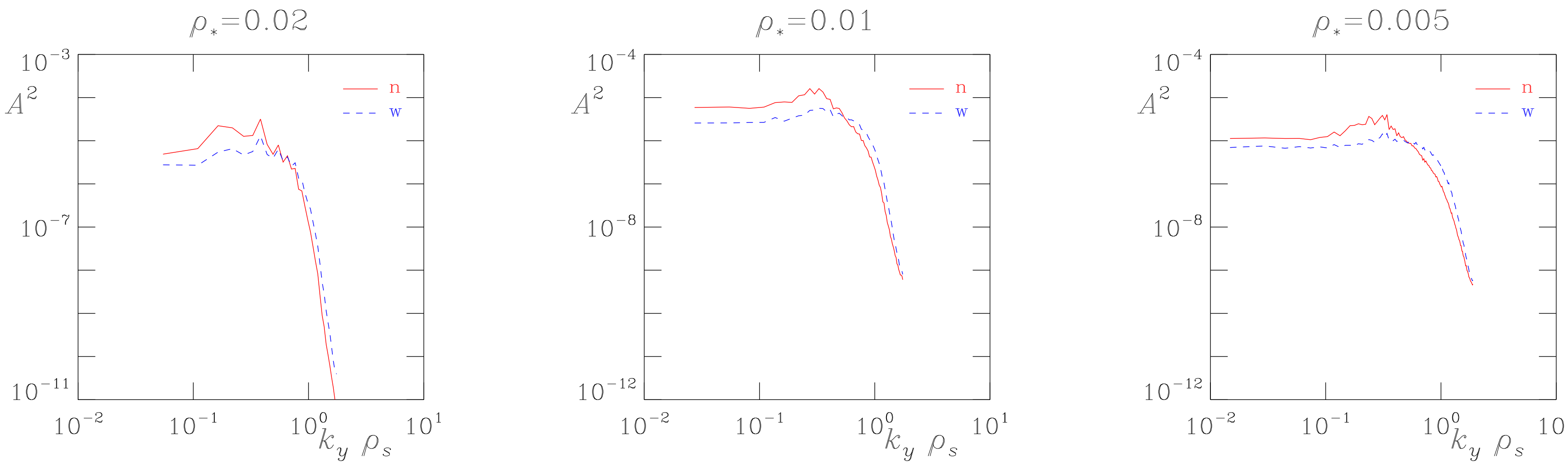
Scale Separation and the Profile Decay Rate

- profile (zonal component ion thermal energy) decay for the three cases

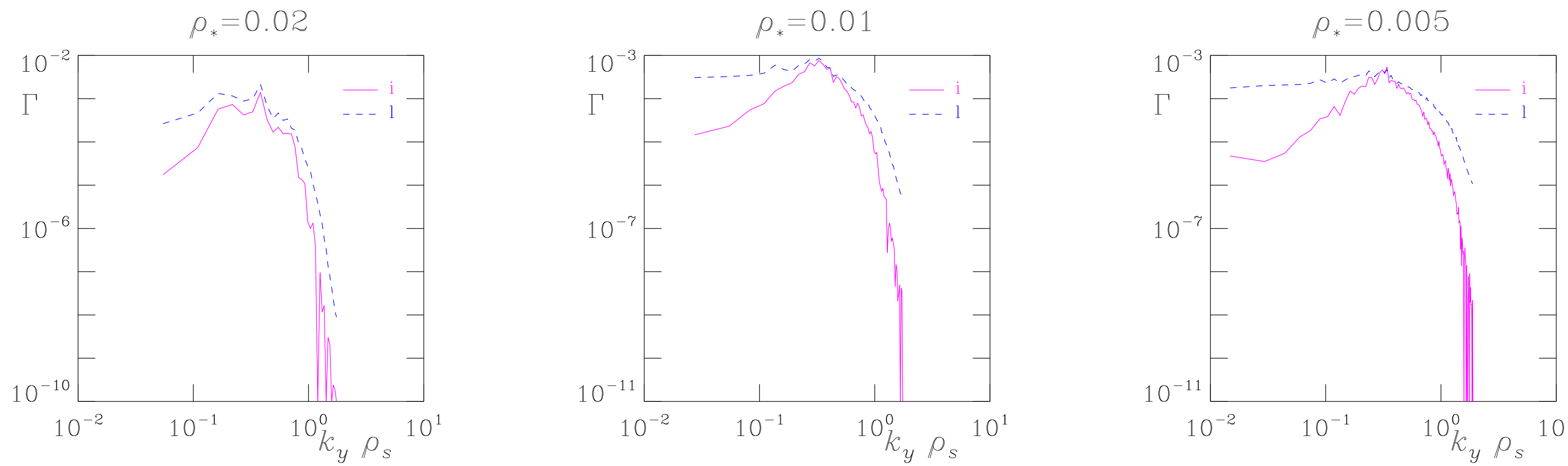


Scale Separation and the Spectrum

- density and vorticity spectra for the three cases



- ion heat source and sink spectra for the three cases



Scale Separation in the Edge

- radial extent is narrow, channeled by finite extend of the region where $c_s/L_\perp > V_e/qR$

$$\hat{\mu} = \frac{m_e}{M_i}\left(\frac{qR}{L_\perp}\right)^2 > 1 \qquad \text{in edge} \qquad\qquad L_x \sim 50, 100 \times \rho_s$$

- extent in drift angle is very large: low $T_e \rightarrow$ large a/ρ_s

$$a \sim 10^3 \times \rho_s \qquad\qquad L_y = 2\pi a/q \sim 2 \times a$$

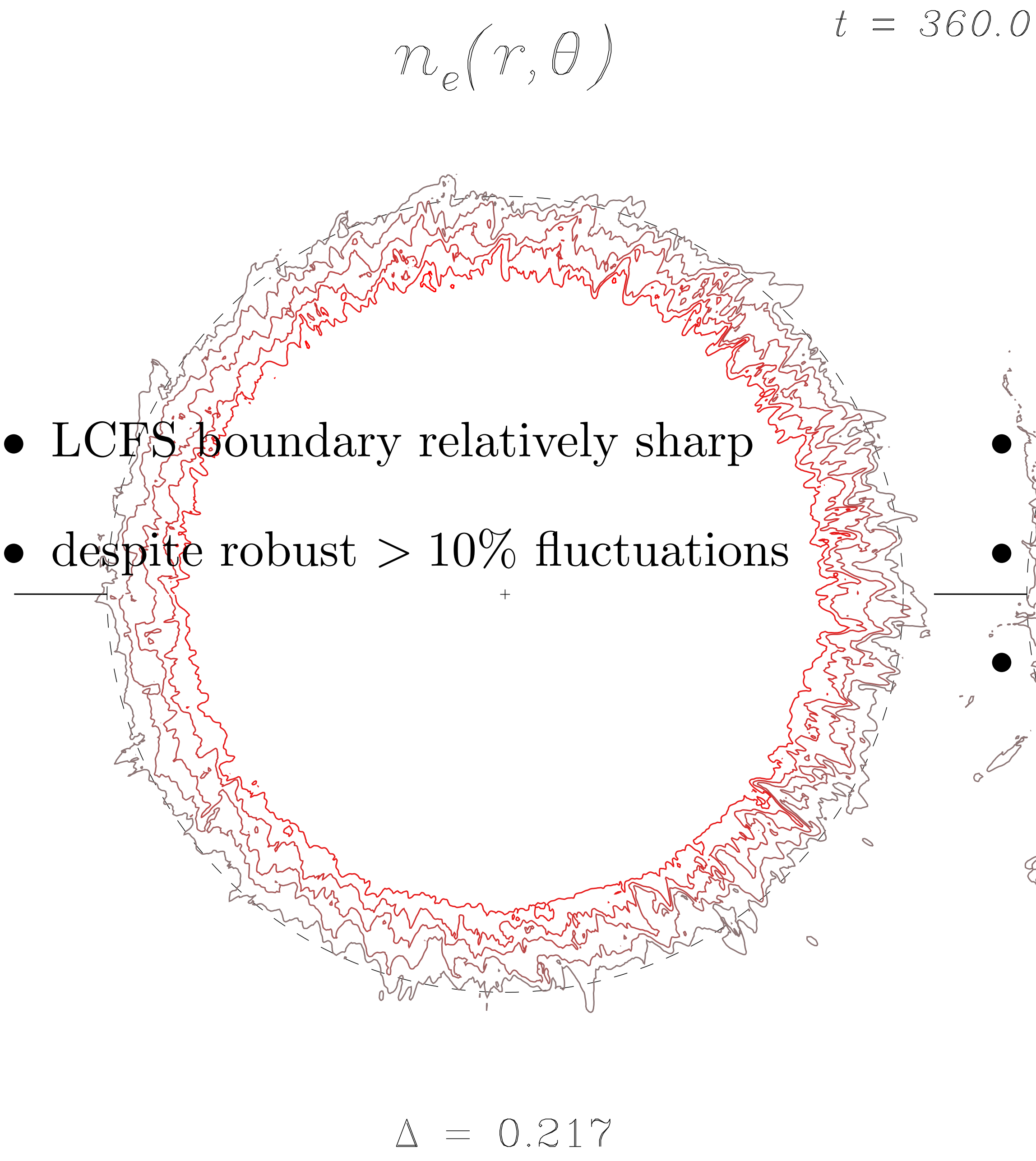
- typical extent for full flux-surface case in medium-sized tokamak

$$L_x = 128\rho_s \qquad\qquad L_y = 2048\rho_s \qquad\qquad L_s = 2\pi qR$$

- typical grid ($2\rho_s$-resolution) (no field aligning: $N_\theta \sim N_\zeta/2 \times \Delta q$ and $16 \rightarrow 2048$)

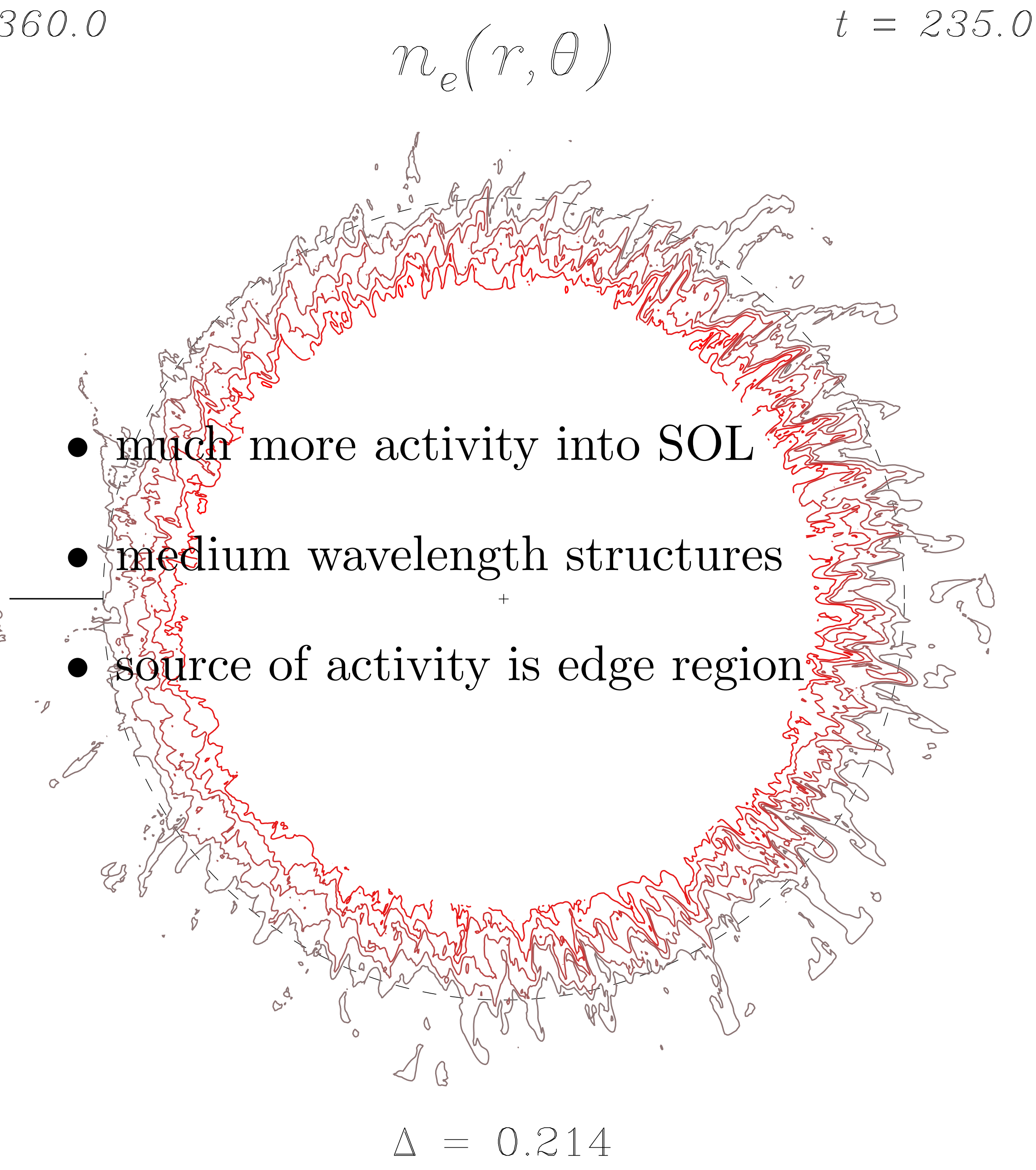
$$N_x \times N_y \times N_s \quad = \quad 64 \times 1024 \times 16$$

Between Bursts



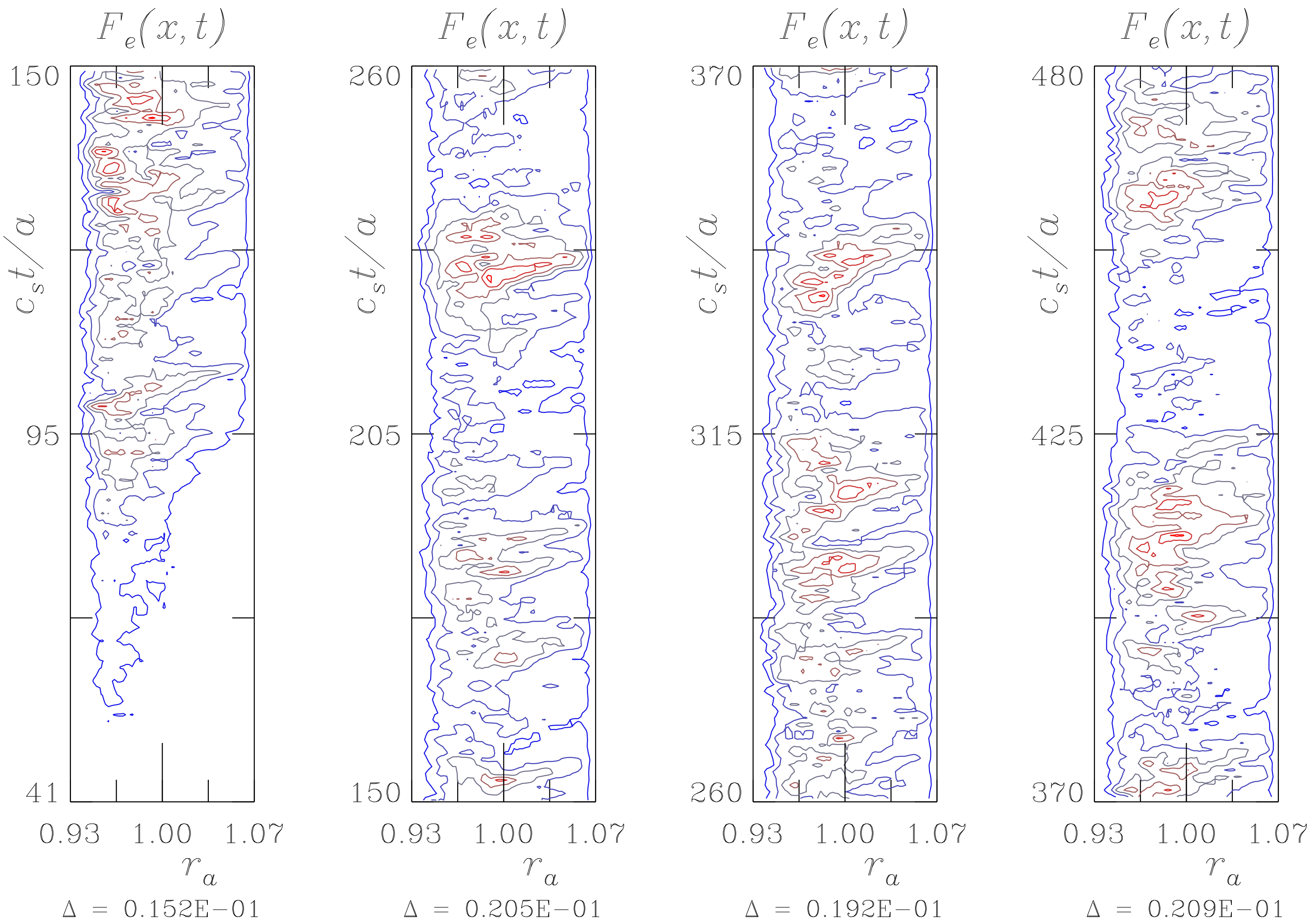
- LCFS boundary relatively sharp
- despite robust > 10% fluctuations

During a Burst



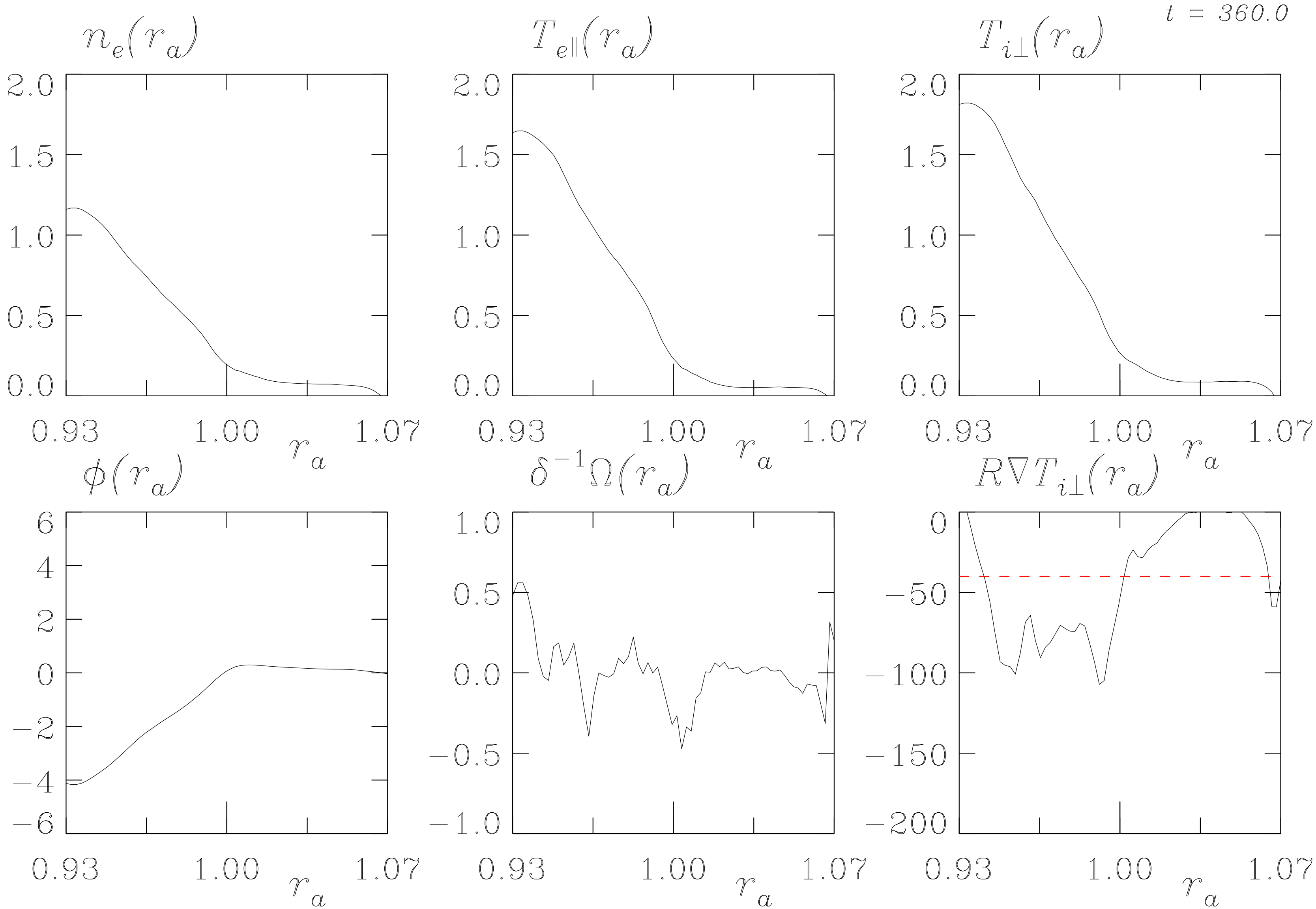
- much more activity into SOL
- medium wavelength structures
- source of activity is edge region

Flux Temporal Behaviour

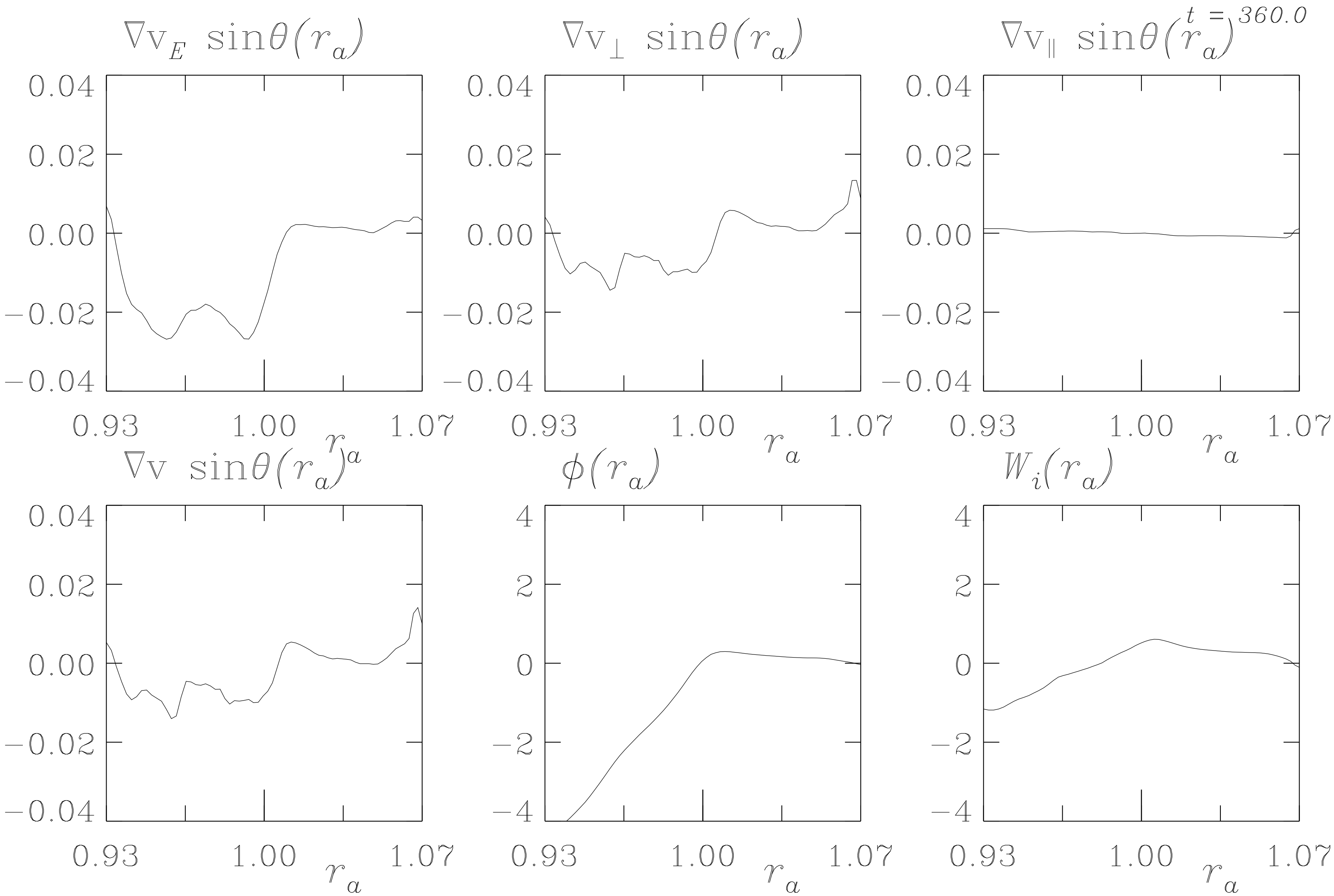


Zonal Profiles between Bursts

- electrostatic potential shows nominal shear layer at LCFS ($r_a = 1$)

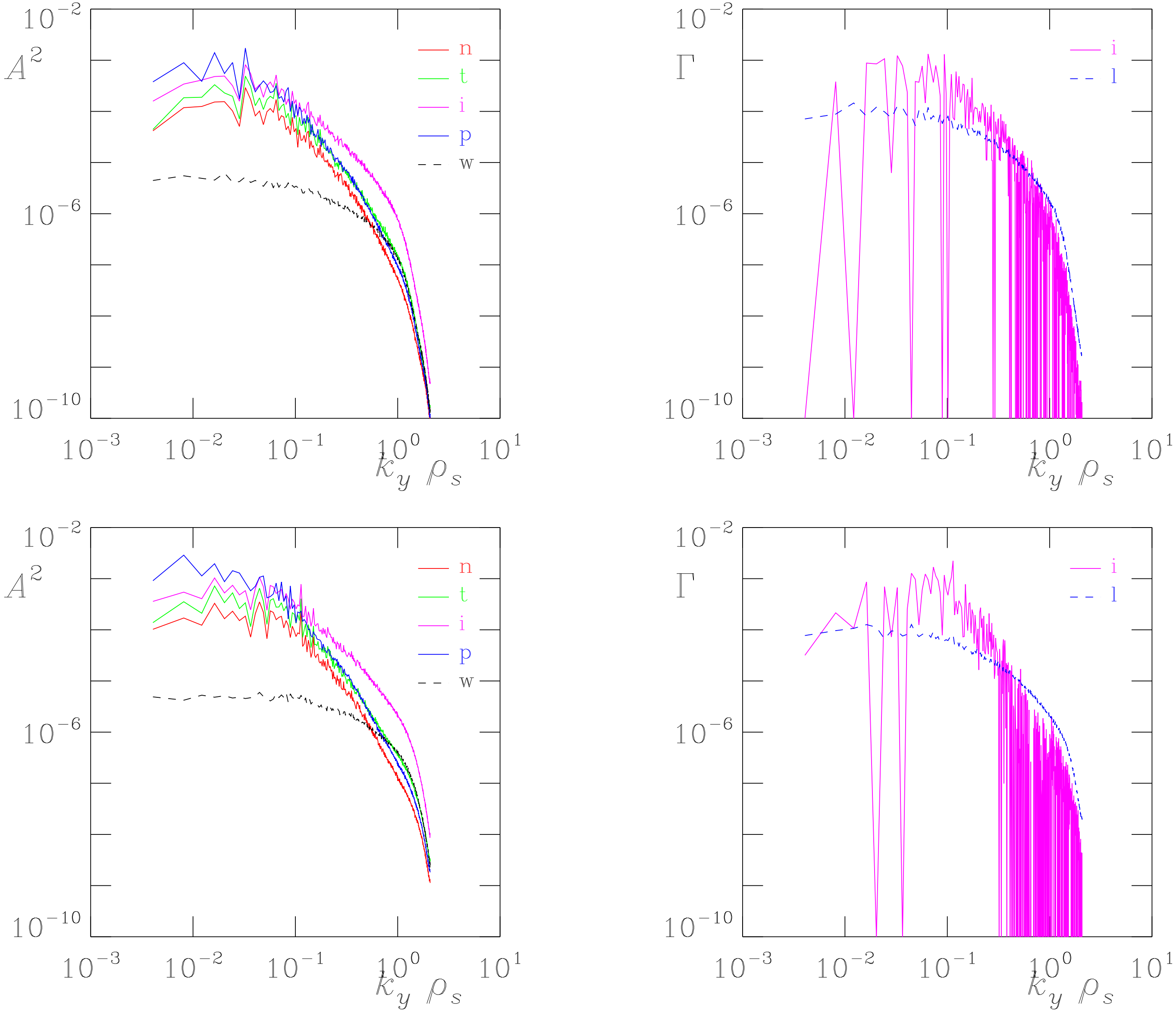


Ion Flow Sideband Divergences between Bursts



Spectra between and during Bursts

- amplitudes/energies (left) and fluxes (right), between (top) and during (bottom)



Burst Notes

- electron/ion heat flux variation a factor of about 3

- bursts are strong events but do not completely destroy the neoclassical equilibrium
  - no "new mode" is involved

- edge/SOL transition is sharp, about 10 to 15ρ_s

- vorticity spectrum always reaches to $k_\perp \rho_i > 1$ since if $T_i \sim T_e$ then $\rho_i \sim \rho_s$

- capture of burst phenomenology requires full scale separation, entire flux surface
  - fluxtube cases give "too clean," too strong bursts (quasiperiodic, factor of 10)

- long-wavelength range $0.01 < k_y \rho_s < 0.1$ necessary as nonlinear energy-dump range

- fluxtube cases can study basic turbulence character
  - but not the self-consistent interaction with neoclassical equilibrium

Edge versus Core

- main parameter differences are ρ_s/L_x and L_y/L_x and R/L_T
  ○ edge: $\hat{\mu} > 1$,   core: $\hat{\mu} < 1$,   following c_s/L_T versus V_e/qR and hence R/L_T (> 50)

- in the edge, electron dynamics is strongly nonadiabatic
  ○ nevertheless, adiabatic coupling is still strong

- hence neither simplified "adiabatic" or "hydrodynamic" or "MHD" models apply

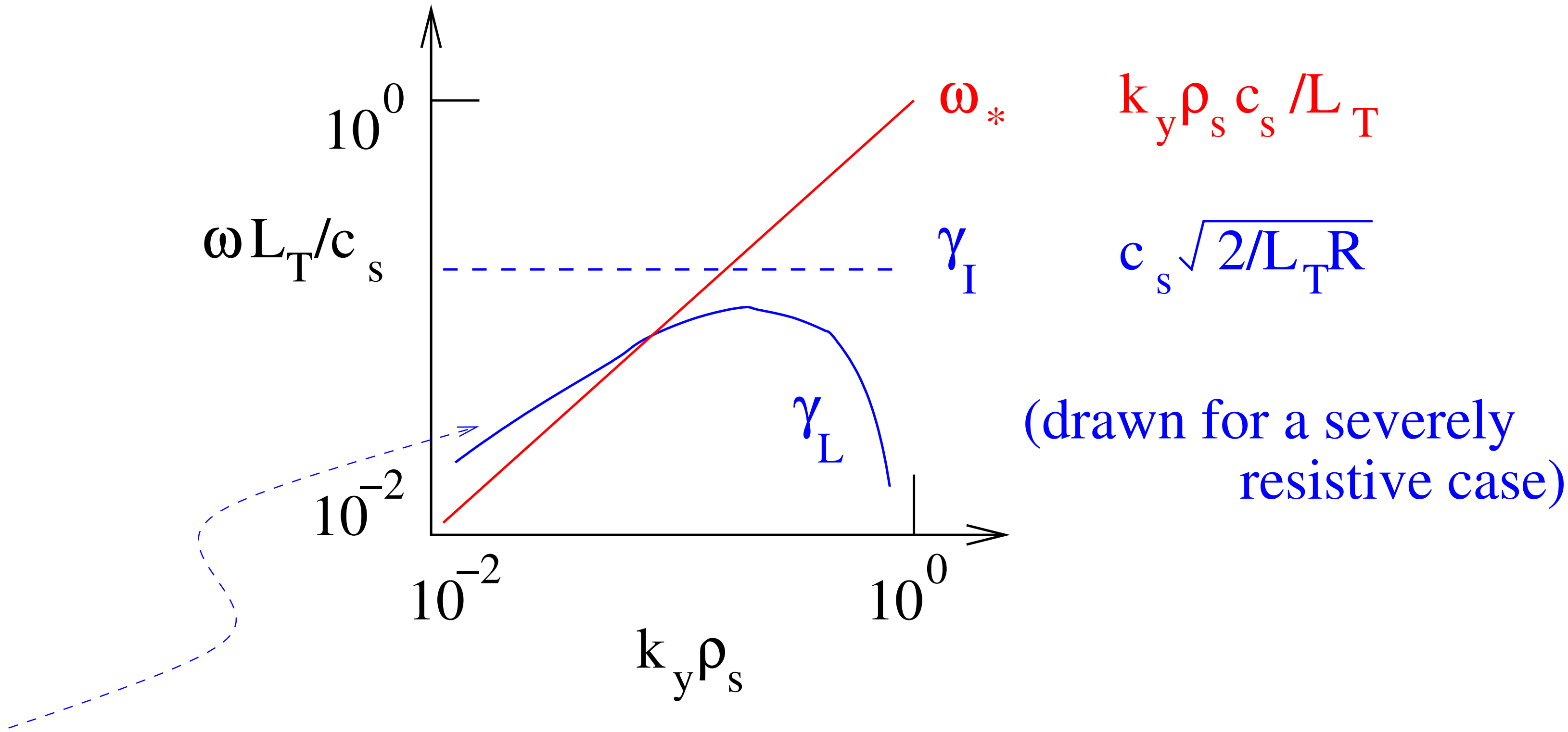
- spectral ranges of free energy, the fluxes, and vorticity separate
  ○ dynamics occupies full spectrum, all scales ρ_s to several L_T are involved

- relevance of underlying nonlinear instability physics
  ○ some strong linear modes are wiped out by native turbulence: $\omega_{\rm rms} > \gamma_L$
  ○ weak long-wave linear modes become important either as sinks or as secondary drive
  (e.g., TAE, reconnection, ballooning)
  ○ a significant fraction of free energy resides in linearly damped modes
  (e.g., dissipative shear Alfvén waves)
  ○ rule of thumb on relevance of instability: $\gamma_L > \omega_*$ for that k_y

- consequences of parameter regime: $k_y\rho_s > \sqrt{L_T/R}$ over most of the drive range

Relevance Range for Linear Instabilities

dispersion space bounded by ideal interchange and diamagnetic rates



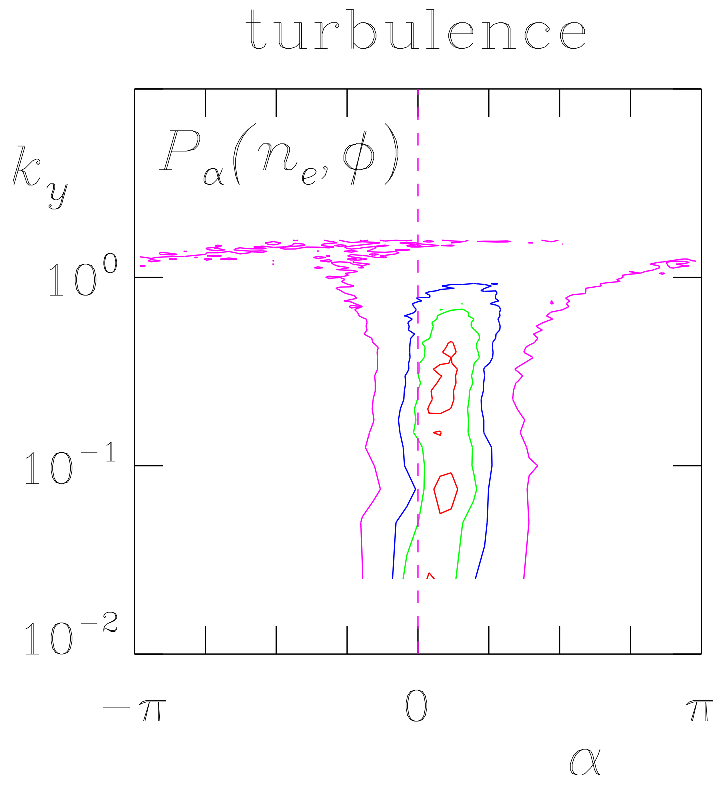
if the linear growth rate is above the red line then the instability is relevant

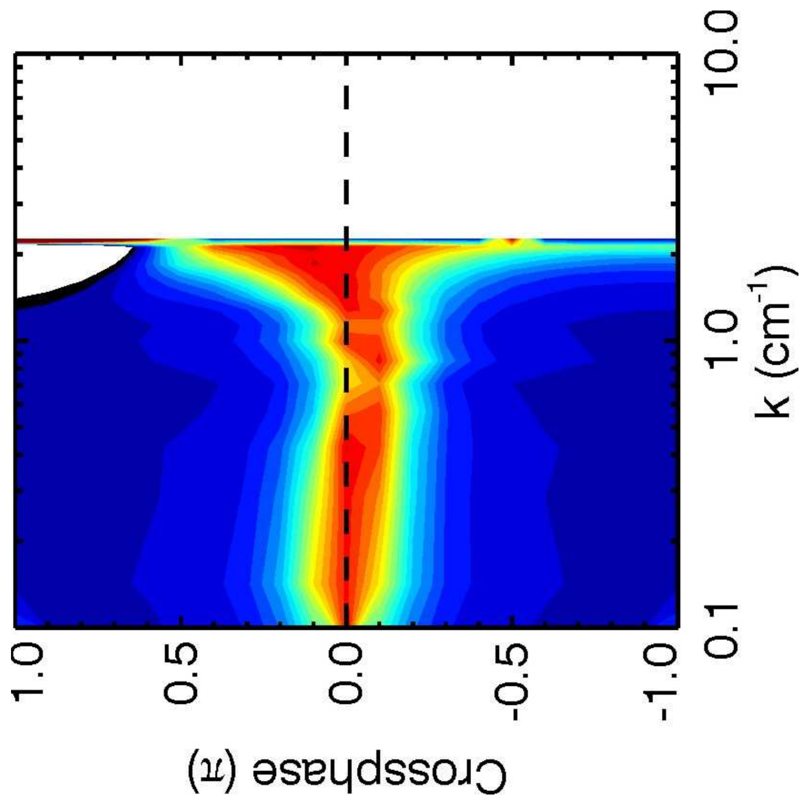
usually, this is not the case anywhere in the spectrum (unless: MHD threshold)

this situation is a direct consequence of very large $R/L_T >> 1$ in the edge

(B Scott New J Phys 2002, Phys Plasmas 2005)

Comparison -- Fluctuation Statistics





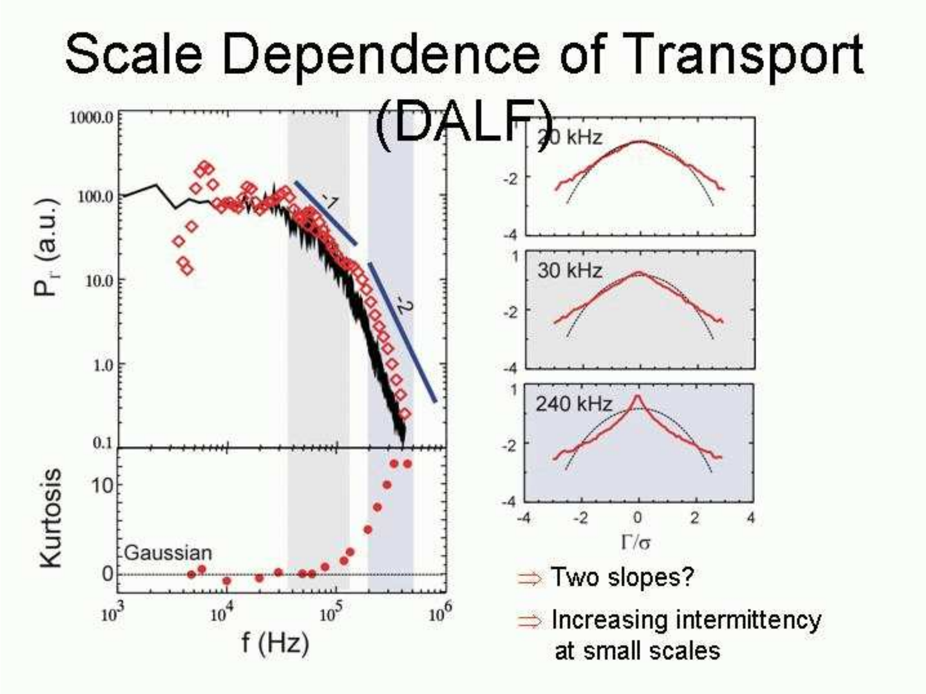
probability distribution of cross phase for each Fourier mode

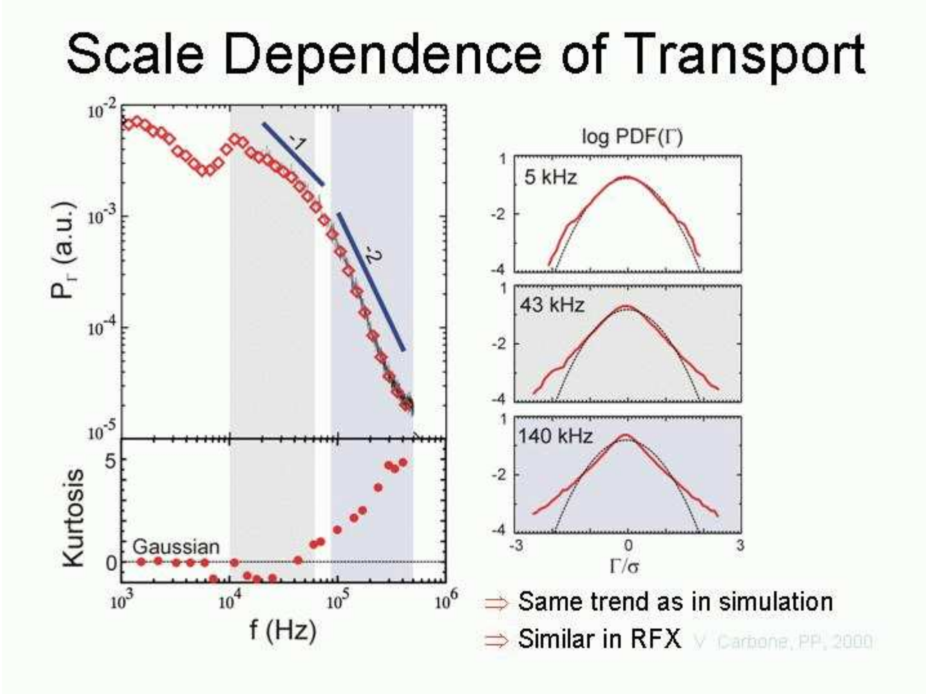
unified spectrum, phase shifts between 0 and $\pi/4$, in code and TJK experiment

basic signature of drift wave mode structure (parallel current dynamics)

(B Scott Plasma Phys Contr Fusion 2003) (U Stroth F Greiner C Lechte et al Phys Plasmas 2004)

Comparison -- Fluctuation Statistics





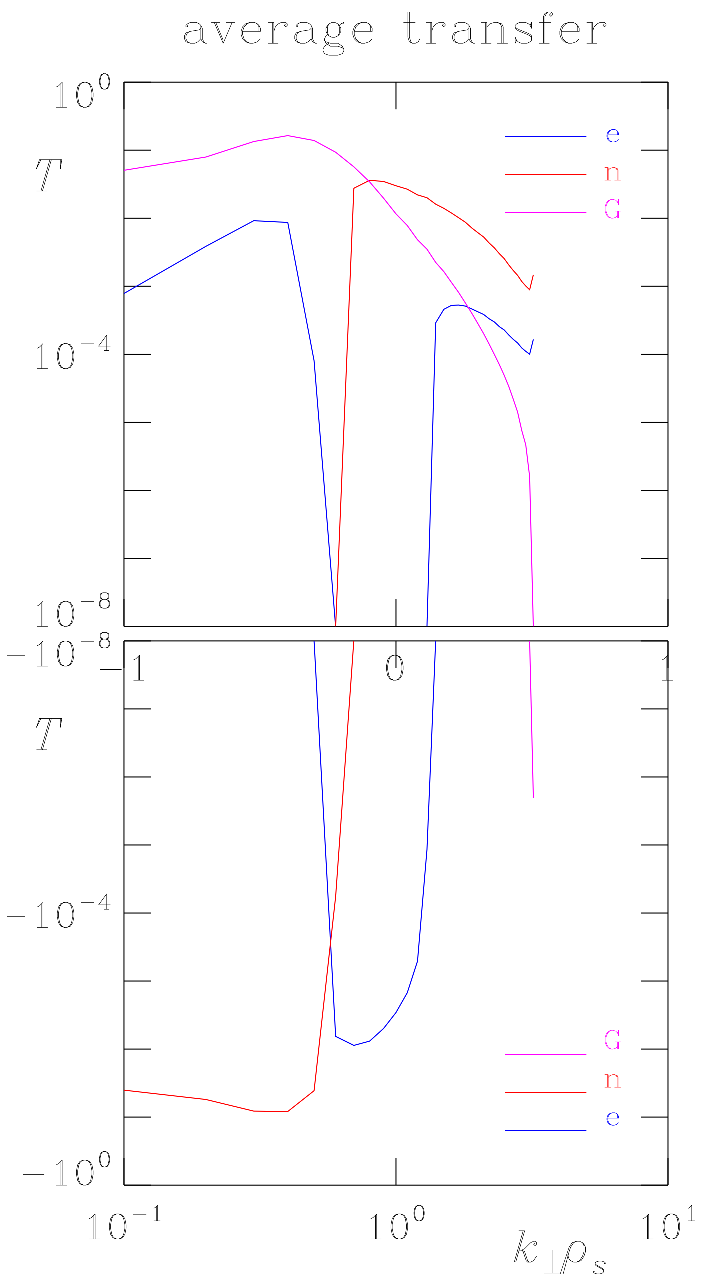
wavelet analysis of fluctuation induced transport in code and TJK experiment

both results show same phenomenology: regime break in spectrum

evidence of nonlinear cascade overcoming drive?

(N Mahdizadeh et al Phys Plasmas 2004)

Nonlinear Free Energy Cascade



direct cascade

--> nonlinear drive at small scales

==> passive scalar regime

frequency/scale correlation

matches with frequency break

evidence for onset of passive scalar regime

The EFDA Integrated Modelling Effort (TF–ITM)

coordinate and establish standards for European codes in all categories

wide effort led by P Strand

Project 4 – instabilities, transport, turbulence

currently: cross–benchmarking on standard cases

global models automatically face the neoclassical equilibrium

separate issues: neoclassical equilibrium, and then transport

currently:

global core benchmarks on Cyclone base case

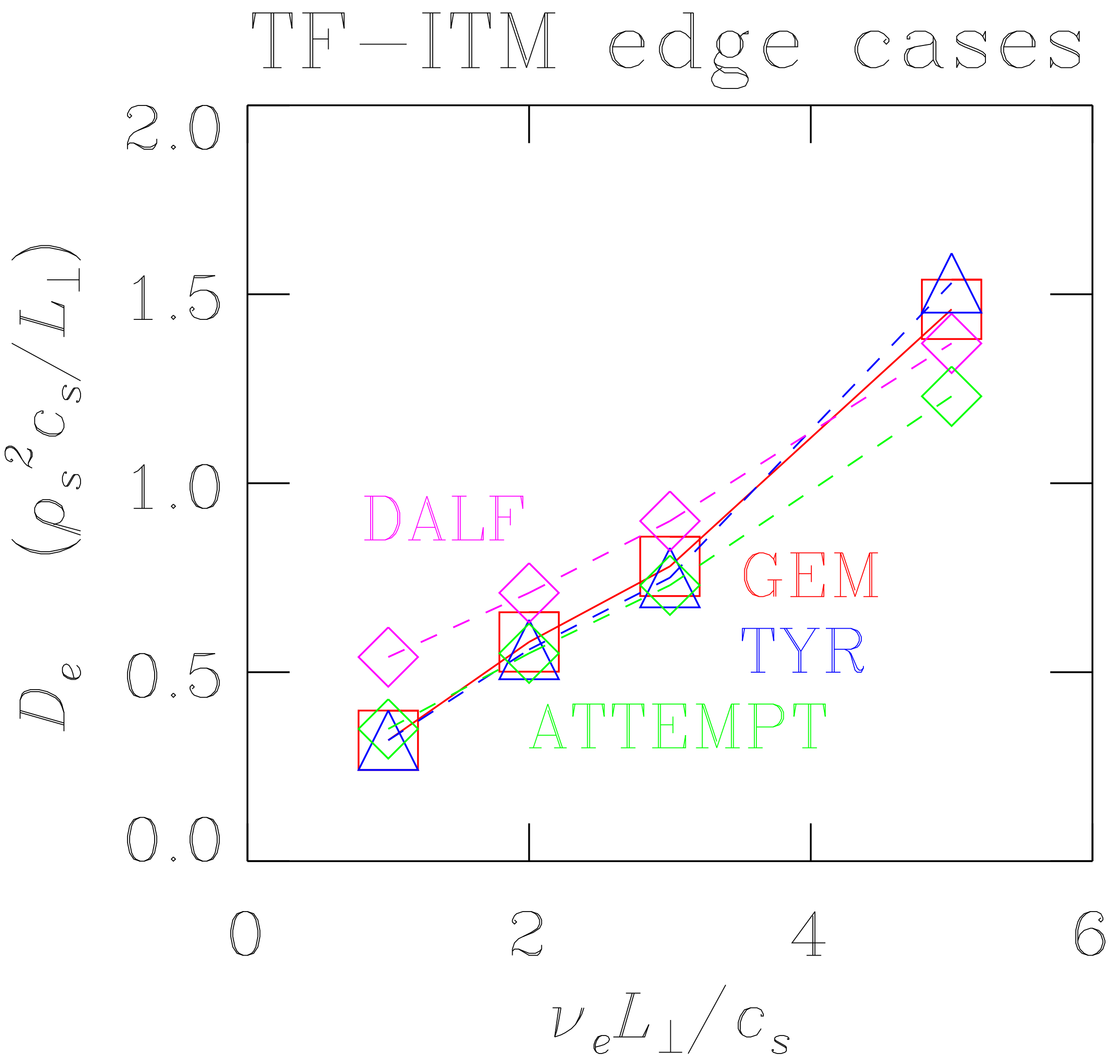
* local and ~~global~~ edge benchmarks on L–mode base case

incorporation of trapping effects in fluid codes (may be hopeless)

local fluid vs gyrofluid drift-Alfvén

edge, collisional, cold-ion electromagnetic, fluxtube, saturated

Risø TYR (blue), Jülich ATTEMPT (green), GEM (red), DALF3 (pink)



Main Points

basics of energetics a central theme for physical understanding

essence of the physics of edge turbulence is nonlinear

scales separate for different parts, linear modes wiped out, character changes

coupling of turbulence to flows extends to the magnetic equilibrium

self consistency: do the magnetic background inside the turbulence model

new physics themes:

* global electromagnetic computation

*** stable reconnection and equilibration currents

incorporation of trapping effects in fluid codes (may be hopeless)

** nonlocal gyrofluid field theory −−> edge/core transition

one should expect surprises affecting design of high performance devices